



Munich Personal RePEc Archive

The Persistence of Inflation in OECD Countries: A Fractionally Integrated Approach

Gadea, Maria and Mayoral, Laura

21. December 2005

Online at <http://mpra.ub.uni-muenchen.de/815/>

MPRA Paper No. 815, posted 07. November 2007 / 01:18

The Persistence of Inflation in OECD Countries: A Fractionally Integrated Approach*

María Dolores Gadea^a and Laura Mayoral^b

^aDepartment of Applied Economics, University of Zaragoza

^bDepartment of Economics and Business, Universitat Pompeu Fabra

The statistical properties of inflation and, in particular, its degree of persistence and stability over time is a subject of intense debate, and no consensus has been achieved yet. The goal of this paper is to analyze this controversy using a general approach, with the aim of providing a plausible explanation for the existing contradictory results. We consider the inflation rates of twenty-one OECD countries which are modeled as fractionally integrated (FI) processes. First, we show analytically that FI can appear in inflation rates after aggregating individual prices from firms that face different costs of adjusting their prices. Then, we provide robust empirical evidence supporting the FI hypothesis using both classical and Bayesian techniques. Next, we estimate impulse response functions and other scalar measures of persistence, achieving an accurate picture of this property and its variation across countries. It is shown that the application of some popular tools for measuring persistence, such as the sum of the AR coefficients, could lead to erroneous conclusions if fractional integration is present. Finally, we explore the existence of changes in inflation inertia using a novel approach. We conclude that the persistence of inflation is very high (although nonpermanent) in most postindustrial countries and that it has remained basically unchanged over the last four decades.

JEL Codes: C22, E31.

*We are grateful to Jordi Galí, Christian Haefke, and Eduardo Ley for their valuable comments. The authors also would like to thank two anonymous referees for helpful comments and the editor for encouragement. We acknowledge financial support from the Spanish Ministry of Education through grants SEC 2003/061006, SEC 2003-04429, and SEC 2003-04476 and also from the Barcelona Economics Program of CREA.

The study of the statistical properties of inflation has attracted a great deal of attention because this variable plays a central role in the design of monetary policy and has important implications for the behavior of private agents. Moreover, new interest in the subject has arisen in the last few years and, as a consequence, a large number of empirical and theoretical papers have appeared recently. Two reasons motivate this upsurge. Firstly, the international monetary context has experienced important changes such as the adoption of inflation-targeting regimes by some countries, the arrival of monetary union in Europe, and a general deflationist process in industrial economies. Secondly, the recent advances in the statistical treatment of time-series data have improved the tools of analysis.

In spite of the great effort, no consensus has been achieved yet about the most appropriate way to model the inflation rate, and various questions remain open. Two fundamental issues emerge in this macroeconomic debate: how to measure the persistence of inflation rates accurately and whether this persistence has changed recently. On the one hand, the degree of inflation persistence is a key element in the monetary transmission mechanism and a determinant of the success of monetary policy in maintaining a stable level of output and inflation simultaneously.¹ On the other hand, detecting whether persistence has fallen recently is crucial in determining the probability of recidivism by the monetary authority (see Sargent 1999) since, as Taylor (1998) and Hall (1999) have pointed out, tests in the spirit of Solow (1968) and Tobin (1968) will tend to reject the hypothesis of monetary neutrality if persistence estimates are revised downward. Thus, understanding the dynamics of inflation is a crucial issue with very important policy implications.

Various economic mechanisms have been put forward to characterize the price formation process, the sticky price models à la Taylor (1979, 1980) and Calvo (1983) being the dominant theoretical background in monetary policy. These models are not completely successful in capturing the observed inflation inertia, so

¹The need to coordinate monetary policy with the degree of inflation persistence has given rise to numerous articles. For instance, Coenen (2003) and Angeloni, Coenen, and Smets (2003) study the robustness of monetary policy when there is uncertainty about the correct persistence of inflation and conclude that it would be preferable to design the monetary target assuming a high inflation inertia.

subsequent modifications have been designed to enhance their empirical performance (e.g., Fuhrer and Moore [1995], Fuhrer [1997], Galí and Gertler [1999], Christiano, Eichenbaum, and Evans [2001], Galí, Gertler, and López-Salido [2001], Roberts [2001], Driscoll and Holden [2004], Coenen and Wieland [2005], etc.). Nevertheless, from a more applied perspective, there is still a lot of controversy about the degree and stability of inflation persistence. On the one hand, there is abundant empirical evidence that postwar inflation exhibits high persistence in industrial countries. The papers of Pivetta and Reis (2004) for the United States and O'Reilly and Whelan (2004) in the euro zone are some examples. On the other hand, it has been argued that the above-mentioned results are very sensitive to the statistical techniques employed and that the observed persistence may be due to the existence of unaccounted structural changes, probably stemming from modifications in the inflation targets of monetary authorities, different exchange rate regimes, or shocks to key prices (see Levin and Piger 2003).² A similar lack of consensus is found in the analysis of persistence stability. Some authors have found evidence of a decrease in inflation inertia in recent years (see Taylor [2000], Cogley and Sargent [2001], and Kim, Nelson, and Piger [2004]) while others, employing different econometric techniques, give support to the opposite conclusion that inflation persistence is better described as unchanged over the last decades (see Batini [2002], Stock [2001], Levin and Piger [2003], O'Reilly and Whelan [2004], and Pivetta and Reis [2004]).

The goal of this article is to shed further light on this controversy by considering a wider statistical framework. Typically, the papers above only consider $I(1)$ or $I(0)$ processes (allowing sometimes for parameter instability) in order to fit these data. Although both formulations can deliver similar short-term predictions if appropriate parameters are chosen, their medium- and long-term implications are drastically different (see Diebold and Senhadji 1996). Processes containing a unit root are characterized by a flat sample autocorrelation function, revealing the fact that the impact of shocks to the series is permanent. In contrast, correlations in $I(0)$ processes decay to zero at an exponential rate, implying that all shocks have a

²It is well known that the existence of changes of regime that are not explicitly taken into account may lead to the detection of spurious persistence (see Perron 1989).

short-lasting effect on the process. It is easy to find situations where this framework can be too restrictive, as there are both economic foundations and empirical evidence suggesting that many macroeconomic and financial variables react to shocks in a different fashion. This is the case, for instance, of variables whose shocks are non-permanent but vanish very slowly (with correlations, if they exist, decaying at a hyperbolic rather than at an exponential rate), resulting in series that may or may not be stationary, in spite of displaying mean reversion.³ To overcome this limitation, a more flexible model has been introduced which is capable of encompassing the $I(1)/I(0)$ paradigm as well as a richer class of persistence behaviors. The autoregressive fractionally integrated moving average (ARFIMA) models are similar to the ARIMA models, but the order of integration, d , is allowed to be any real number instead of only integer ones. It turns out that the former models are very convenient for analyzing the persistence properties of inflation since they are able to account for a wide variety of persistence features very parsimoniously.

In this paper, we demonstrate that fractionally integrated (FI) behavior can appear in the inflation rate as a result of aggregating prices from firms that are heterogeneous in their price adjustment costs, and we test this conjecture on a large data set containing twenty-one OECD countries.⁴ In order to do so, FI models are estimated and tested against other popular specifications (such as different ARMA and ARIMA models, possibly affected by parameter instability) using both classical and Bayesian techniques.

We have found strong support for our conjecture, which is robust across the different countries, the various competing models, and the set of employed techniques. According to these results, it is shown that if ARIMA models are used to measure persistence, they will

³Evidence of these features has been found in variables such as GNP (Diebold and Rudebusch [1989] and Sowell [1992b]); asset price and exchange rate volatility (Andersen and Bollerslev [1997], Andersen et al. [1999], Ding, Granger, and Engle [1993], and Breidt, Crato, and Lima [1998]); political opinion data (Byers, Davidson, and Peel [1997]); and many others. See Henry and Zaffaroni (2002) for other significant references.

⁴FI models have already been employed in the literature to model inflation data, but, to the best of our knowledge, no economic justification for the presence of FI has been provided. See Baillie, Chung, and Tieslau (1992, 1996), Hassler and Wolters (1995), Franses and Ooms (1997), Barkoulas, Baum, and Oguz (1998), Bos, Franses, and Ooms (1999, 2002), Delgado and Robinson (1994), Baum, Barkoulas, and Caglayan (1999), and Ooms and Doornik (1999).

tend to overestimate this property. Furthermore, we show that the usual procedure of fitting an $AR(k)$ process to the data and identifying a value of the sum of the AR coefficients close to one with the existence of an (integer) unit root can easily lead to persistence overestimation. This is so because any FI model with a fractional integration order strictly greater than zero admits an $AR(\infty)$ representation that verifies that the sum of the corresponding coefficients ($\rho(1)$) is equal to one.⁵ When fitting an AR model to an FI process, any sensible information criterion chooses a finite and relatively small value of k , but the sum of the estimated coefficients is still close to one in most cases. Therefore, prudence recommends to interpret $\rho(1) \approx 1$ not as a signal of an integer unit root but just as an indication of some type of integration, possibly fractional, in the data. The implications in term of persistence of the former or the latter interpretation are drastically different.⁶

The main results that we have obtained can be summarized as follows. Once fractional integration is allowed for, both the $I(0)$ and the $I(1)$ specifications are clearly rejected. Furthermore, for most countries the FI specification is also preferred to the alternative of $I(0)$ processes suffering from parameter instability, which could be an alternative explanation of the observed persistence.⁷ Inflation rates are estimated using different techniques, and it is shown that they are best characterized as FI models with a memory parameter, d , around 0.6–0.8. This implies that they are very persistent, nonstationary; however, as opposed to $I(1)$ variables, shocks have a non-permanent character, so the series are mean reverting. We provide various persistence measures that permit an adequate comparison of inflation inertia across countries and their evolution over time. We

⁵This is true for the same reasons as in the $I(1)$ case: the polynomial of the AR expansion contains the factor $(1 - L)^d$, where L is the lag operator and d is a real number representing the order of integration. Clearly, $L = 1$ is a root of this polynomial if $d > 0$ which, in turn, implies that the sum of the AR coefficients associated with lagged values of the process has to be equal to one. See section 4 for a more technical explanation.

⁶As it will be shown in section 2, the class of FI models with an integration order, d , strictly greater than zero is very large, containing both stationary and nonstationary processes that, in the latter case, may or may not be mean reverting.

⁷It is well known that FI models and $I(0)$ processes with structural changes may look very similar (see section 3). The possibility of directly testing these hypotheses is also a major novelty of this paper.

find important differences across countries. According to the half-life measure (HL), U.S. inflation is the most persistent and inflation of Central and Nordic European countries presents the lowest degree of inertia. We also provide persistence estimates computed from ARIMA specifications and show that the permanent-shock restriction introduced by the unit-root hypothesis leads to persistence overestimation. Finally, we have also explored the possibility of a change in persistence, but for most countries we find no evidence of any such change. Throughout the article, our results are compared with those of previous works, and explanations of the divergence are provided. We also describe some potential pitfalls deriving from the use of some popular persistence tools when the DGP is FI but this property is not taken into account.

The rest of the paper is structured as follows. Section 1 presents a standard preliminary analysis of inflation. Section 2 describes the concept and the main characteristics of fractionally integrated processes and provides an economic explanation of the existence of these features in inflation data. Section 3 reports the results of fitting ARFIMA models to this data set by using both classical and Bayesian methods and tests the $FI(d)$ hypothesis against various alternatives such as $I(1)$, $I(0)$, and $I(0)$ with a structural break in the mean. Impulse response functions and other scalar measures of persistence are provided in section 4. Section 5 analyzes the hypothesis of a change in inflation persistence. Finally, section 6 gives some concluding remarks.

1. Data Description and Preliminary Tests

We consider the quarterly consumer price index in the period running from the first quarter of 1957 to the last quarter of 2003 for twenty-one OECD countries. The data have been obtained from the International Financial Statistics database of the International Monetary Fund. The countries included in the study are Australia (AU), Austria (AUS), Belgium (BE), Canada (CA), Denmark (DK), Finland (FI), France (FR), Germany (GE), Greece (GR), Italy (IT), Japan (JP), Luxembourg (LX), Netherlands (NL), New Zealand (NZ), Norway (NO), Portugal (PO), Spain (SP), Sweden (SWE), Switzerland (SWI), United Kingdom (UK), and the United States (USA).

In order to construct the inflation rates, we have proceeded as follows. Firstly, the price series for each country has been seasonally adjusted using the X12 quarterly seasonal adjustment method of the U.S. Census Bureau. Secondly, inflation rates are computed as $\pi_t^i = \ln P_t^i - \ln P_{t-1}^i$ and, finally, an outlier analysis has been carried out and the additive outliers (AO) that clashed with methodological changes in the price indices have been removed. This has been the case of Austria (1957:3), Belgium (1967:1, 1971:1), Finland (1972:1), France (1980:1), Germany (1991:1), Greece (1959:1, 1970:1), Italy (1967:1), Netherlands (1960:1, 1961:1, 1981:1, 1984:2), New Zealand (1970:1), and Sweden (1980:1).

The evolution of the inflation series is shown in figures 5 to 7 (see the appendix). The well-known trends of postwar inflation in developed countries can be easily identified in these graphs. Starting from low levels in the 1960s, around 3 percent for most countries, prices rose dramatically in the 1970s after the oil crisis (inflation figures almost tripled) and this sharp increase was accompanied by high volatility. In the 1980s, inflation was moderately reduced by the application of tight monetary policies, but high levels of volatility were still observed. Finally, the 1990s were characterized by a generalized decrease in the mean and in the variance of inflation.

The preliminary analysis proceeds as follows. Firstly, standard unit-root tests have been computed on the inflation series and the results are presented in table 1. To be precise, the ADF test of Dickey and Fuller (1981), the PP of Phillips and Perron (1988), the MZ-GLS of Ng and Perron (2001), and the KPSS of Kwiatkowski et al. (1992) have been employed. Columns 1–3 of table 1 take the $I(1)$ model as the null hypothesis, whereas the fourth column considers the $I(0)$. The latter hypothesis is clearly rejected for all countries at the 1 percent significance level (column 4), whereas the $I(1)$ is rejected for sixteen out of the twenty-one countries by at least two tests (columns 1–3). Four countries (IT, SP, PO, and USA) present rejection in one of the tests, and for only one country (BE) is it not possible to reject the $I(1)$ conjecture with any of these tests. Since unit-root tests are known to lack power in many relevant situations, the results above cast serious doubts about the existence of a unit root in inflation rates. This finding is relevant because some tests (like the monetary neutrality tests) start by assuming a unit root in inflation rates and are not valid outside this framework.

Table 1. Unit-Root and Stationarity Tests

	ADF	PP	MZ _t -GLS	KPSS
AU	-2.39 (2)	-4.46** (8)	-2.16* (2)	0.88** (10)
AUS	-4.71** (2)	-5.60** (8)	-0.35 (2)	0.94** (9)
BE	-2.26 (3)	-2.77 (10)	-1.85 (3)	0.79** (10)
CA	-3.01* (1)	-3.89** (3)	-2.77** (1)	0.93** (10)
DK	-3.49** (2)	-4.94** (8)	-1.62 (2)	1.38** (9)
FI	-3.32* (1)	-4.11** (5)	-3.06** (1)	1.34** (10)
FR	-3.69** (1)	-3.49** (6)	-3.22** (1)	1.46** (10)
GE	-3.01* (2)	-4.75** (7)	-2.77** (2)	0.70** (10)
GR	-3.23* (1)	-3.71** (2)	-2.80** (1)	1.32** (10)
IT	-1.50 (5)	-3.56** (10)	-0.91 (5)	0.90** (10)
JP	-2.76 (2)	-4.60** (6)	-2.50* (2)	1.78** (10)
LX	-3.11* (7)	-4.38** (4)	-3.62** (7)	0.72** (10)
NL	-3.81** (3)	-5.20** (7)	-3.32** (3)	1.01** (9)
NZ	-4.14** (1)	-4.42** (6)	-3.42** (1)	1.00** (10)
NO	-3.42* (1)	-2.77 (1)	3.16** (1)	0.99** (10)
PO	-2.02 (4)	-3.74** (2)	-1.42 (4)	1.01** (10)
SP	-2.19 (4)	-5.08** (3)	-1.94 (4)	1.13** (10)
SWE	-3.00* (2)	-5.38** (7)	-2.28* (2)	1.04** (10)
SWI	-3.08* (2)	-5.09** (4)	-2.84** (2)	0.82** (10)
UK	-3.22* (1)	-3.26* (3)	-2.89** (1)	0.85** (10)
USA	-2.61 (3)	-2.63 (4)	-2.70** (3)	0.75** (10)

Notes: **, * denote significance at the 1 percent and 5 percent level, respectively. Figures in parentheses correspond to the number of lags and the bandwidth for the ADF and MZ_t-GLS and the PP and KPSS, respectively. Lag length was chosen according to the SBIC criterion. Bartlett's window was used as a kernel estimator in the PP and KPSS (bandwidth was chosen according to Newey and West 1994).

To sum up, since for most countries both the $I(0)$ and the $I(1)$ hypotheses are rejected, it seems that the ARIMA framework does not provide a good characterization of this data set. This result has been interpreted in the literature as an indicator of a behavior midway between the $I(0)$ and the $I(1)$ formulations.⁸ If a process is $I(1)$, all shocks have a permanent effect, whereas they disappear exponentially when the process is $I(0)$. An alternative to both formulations that has been widely explored in the literature is the existence of structural breaks. This amounts to considering that only a few shocks, such as stock market crashes, oil crises, wars, etc., have a permanent effect on the series while all the others vanish rapidly. Perron (1989) showed that standard unit-root tests are not able to reject the $I(1)$ hypothesis if a trend stationary process suffers from occasional breaks in the parameters that describe the trend and/or the level.

To explore the existence of breaks in the mean, we employ the method proposed by Bai and Perron (1998, 2003a, 2003b), henceforth BP, for multiple structural breaks. BP propose three types of tests. The $\text{supF}_T(k)$ test considers the null hypothesis of no breaks against the alternative of k breaks. The $\text{supF}_T(l+1/l)$ test takes the existence of l breaks, with $l = 0, 1, \dots$, as H_0 against the alternative of $l+1$ changes. Finally, the so-called “double maximum” tests, UDmax and WDmax, test the null of absence of structural breaks versus the existence of an unknown number of breaks. Bai and Perron (2003a) suggest beginning with the sequential test $\text{supF}_T(l+1/l)$. If no break is detected, they recommend checking this result with the UDmax and WDmax tests to see if at least one break exists. When this is the case, they recommend continuing with a sequential application of the $\text{supF}_T(l+1/l)$ test, with $l = 1, \dots$. This strategy has been followed to obtain the figures in table 2.

To test the changes in the level of the series, the following representation has been considered:

$$\pi_t^i = \varphi + \varsigma_t^i,$$

⁸It is well known that standard unit roots still have power when the DGP is not the one postulated under the alternative hypothesis. This is the case, for instance, of fractionally integrated processes (see Diebold and Rudebusch [1991] and Lee and Schmidt [1996] for the DF and KPSS tests, respectively) or some types of structural breaks (see Perron 1989).

Table 2. Breaks in the Mean

	Number of Breaks	Dates of the Breaks
AU	2	1970:4, 1991:1
AUS	3	1970:1, 1983:3, 1995:4
BE	2	1971:4, 1985:3
CA	4	1965:1, 1972:3, 1983:1, 1990:4
DK	3	1972:4, 1985:2, 1992:1
FI	3	1971:1, 1982:3, 1991:2
FR	3	1973:2, 1985:3, 1992:3
GE	2	1970:1, 1983:1
GR	2	1973:1, 1993:3
IT	3	1972:2, 1983:3, 1995:3
JP	2	1981:3, 1993:4
LX	2	1970:1, 1985:3
NL	2	1963:4, 1985:4
NZ	2	1970:1, 1988:3
NO	2	1970:4, 1990:3
PO	4	1963:4, 1971:2, 1983:5, 1992:3
SP	4	1973:2, 1980:1, 1986:4, 1995:3
SWE	2	1970:1, 1992:1
SWI	1	1993:3
UK	3	1970:1, 1991:1, 1982:1
USA	2	1967:3, 1982:4
Note: The consistent covariance matrix is constructed using a quadratic kernel following Andrews (1991).		

where φ is a constant capturing the level of the series and ς_t^i is a (short-memory) linear process. Following Perron (1989), attention is focused on sharp changes of the level, φ . A maximum number of five breaks has been considered, which, in accordance with the sample size $T = 186$, supposes a trimming $\varepsilon = 0.15$. The process ς_t^i is allowed to present autocorrelation and heteroskedasticity. A nonparametric correction has been employed to take account of these effects.

The results of applying the multiple-break tests to changes in the level of the inflation rates are presented in table 2. For most countries two or three breaks in the level are detected. The first

break usually takes place at the beginning of the 1970s, whereas the second is located in the middle of the 1980s. The third, if it exists, occurs at the beginning of the 1990s. Thus, the chronology of the break points is in agreement with the general features of inflation discussed above.

The preliminary analysis of the inflation processes of OECD countries highlights the difficulties of modeling these series. On the one hand, there is evidence against both short-memory stationarity ($I(0)$) and unit-root behavior, which are the most common formulations employed to model these series. An alternative to both settings is to consider a model containing structural breaks in some parameters, and evidence supporting this hypothesis has been found. If the latter were true, it would mean that the persistence often found in these series is likely to be spurious. This is the conclusion put forward by Levin and Piger (2003). They analyze the inflation rates of twelve industrial countries and find evidence of breaks in the intercept of the inflation rate. They claim that conditional on these breaks, many countries do not show strong persistence.

Nevertheless, the existence of structural breaks is not the only alternative to the $I(0)/I(1)$ framework. Fractionally integrated models can also bridge the gap between these two formulations. Moreover, it is well-known that FI and structural breaks can be easily confused. Since both types of models have very different implications in terms of persistence, it is crucial to determine which of the two phenomena is more likely to be present in the data. Sections 2 and 3 will deal with this issue.

2. Fractional Integration in Inflation Data

The previous results cast serious doubts on the adequacy of either the $I(1)$ or the $I(0)$ models to fit inflation series. When one is interested in analyzing the long-run impact of contemporaneous shocks, the above categories represent two extreme possibilities. Models containing a unit root are characterized by shocks that have a permanent effect, while innovations of $I(0)$ processes disappear so fast that correlations decay at an exponential rate. Nevertheless, it has been shown that this framework could be too narrow in many instances, as there is ample empirical evidence suggesting that shocks of many macroeconomic and financial series behave differently. A class that

embeds both the I(1) and the I(0) models and, at the same time, is able to account for richer persistence types is given by the so-called fractionally integrated (FI) models. Among this class, the most popular parametric model is the ARFIMA one, independently introduced by Granger and Joyeux (1980) and Hosking (1981). The main advantage of this formulation with respect to the ARIMA one is the introduction of a new parameter, d , that models the “memory” of the process, that is, the medium- and long-run impact of shocks on the process. More specifically, y_t is an ARFIMA(p, d, q) if it can be written as

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t, \varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2),$$

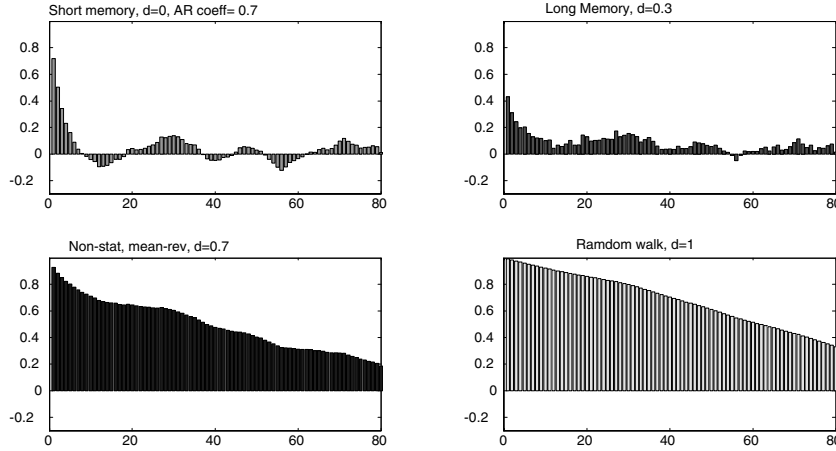
where the so-called *memory parameter*, d , determines the integration order of the series and is allowed to take values in the real, as opposed to the integer, set of numbers.⁹ The terms $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - \dots - \theta_q L^q$ represent the autoregressive and moving average polynomials, respectively, with all their roots lying outside the unit circle. While d captures the medium- and long-run behavior of the process, $\Phi(L)$ and $\Theta(L)$ model the short-run dynamics. As Diebold and Rudebusch (1989) notice, this provides for “parsimonious and flexible modeling of low frequency variation.”¹⁰

The bigger the value of d , the more persistent the process is. Stationarity and invertibility require $|d| < 1/2$, which can always be achieved by taking a suitable number of (integer) differences. *Short memory* is implied by a value of $d = 0$, where the process is characterized by absolutely summable correlations decaying at an exponential rate. By contrast, *long memory* occurs whenever d belongs to the (0,0.5) interval. Hosking (1981) showed that the correlation function in this case is proportional to k^{2d-1} as $k \rightarrow \infty$, that is, it decays at a hyperbolic rather than at an exponential rate. These processes are also characterized by an unbounded spectral density at frequency zero. These facts reflect the slower decay of shocks with

⁹ARIMA models are a particular case, where $d = 0, 1, 2, \dots$. Notice that, in contrast to the ARIMA case, in the ARFIMA framework, d is a parameter that requires estimation.

¹⁰Furthermore, the fact of having two sets of parameters modeling the long- and short-run dynamics separately avoids some estimation problems that might affect the ARMA processes. As Sowell (1992b) points out, maximum likelihood estimation of ARMA models may sacrifice the long-run fit to obtain a better fit of the short-run behavior.

Figure 1. Sample Autocorrelation Function of Several Processes



respect to the $I(0)$ case. A particularly interesting region for macroeconomic applications is the interval $d \in [0.5, 1)$. In this range, shocks are transitory, but the impulse response to shocks vanishes so slowly that the variance is not bounded and, therefore, the process is non-stationary in spite of being mean reverting (as shocks eventually disappear). Shocks have a permanent effect whenever $d \geq 1$.

Figure 1 illustrates the differences described above. The main diagonal contains the sample correlation function up to lag 80 of an $I(0)$ and an $I(1)$ process, respectively, whereas the other diagonal represents the same function for two FI processes. It can be seen that, after a few lags, the $I(0)$ and the $I(1)$ characterizations are drastically different, while the FI ones are able to fill the gap between the former models. The upper left graph depicts the sample autocorrelation function of an $AR(1)$ process with an autoregressive coefficient equal to 0.7. Although this process is highly correlated at first lags, autocorrelations decay to zero very fast and become non-significant after a few lags. The behavior changes drastically whenever d is allowed to take strictly positive values. The long-memory case is illustrated in the upper right graph that contains the sample correlation function of an $ARFIMA(0, 0.3, 0)$. It is characterized by a slow decay of correlations, which remain significantly different from zero even at distant horizons. The two bottom graphs represent an

ARFIMA(0,0.7,0) and an I(1) process. Both are nonstationary, very persistent, but correlations for the former decay faster, revealing the fact that the process is eventually mean reverting. The graph on the lower right corresponds to a random walk where all shocks have a permanent effect.

The success of these models in economics may be attributed to the development of a rationale for the presence of FI in macro-level economic and financial systems. Robinson (1978) and Granger (1980) showed that FI behavior could appear in the aggregate produced from a large number of heterogeneous I(0) processes describing the microeconomic dynamics of each unit. This result has been incorporated in different economic settings to show analytically that some relevant variables can display FI¹¹ and is also the approach that we exploit to justify the existence of FI behavior in the inflation rate. Another way of obtaining FI behavior was proposed by Parke (1999). He considers the cumulation of a sequence of shocks that switch to zero after a random delay. If the probability that a shock survives for k periods, p_k , decreases with k at the rate $p_k = k^{2d-2}$ for $d \in (0, 1]$, Parke demonstrates that the error duration model generates a process with the same autocovariance structure as an I(d) process. He also shows how this mechanism can be applied to generate FI in aggregate employment and asset price volatility. From an empirical point of view, evidence supporting FI in financial and macroeconomic data is very large. See Henry and Zaffaroni (2002) for a detailed list of references.

Operationally, a binomial expansion of the operator $(1 - L)^d$ is used in order to fractionally differentiate a time series:

$$(1 - L)^d = \sum_{i=0}^{\infty} \pi_i(d) L^i, \quad (1)$$

where

$$\pi_i = \Gamma(i - d) / \Gamma(-d) \Gamma(i + 1) \quad (2)$$

and $\Gamma(\cdot)$ denotes the gamma function. When $d = 1$, (1) is just the usual first-differencing filter. For noninteger d , the operator $(1 - L)^d$ is an infinite-order lag-operator polynomial with coefficients that decay

¹¹Some examples are Michelacci and Zaffaroni (2000), Abadir and Talmain (2002), Haubrich and Lo (2001), Byers, Davidson, and Peel (1997), etc.

very slowly. Since the expansion is infinite, a truncation is needed in order to fractionally differentiate a series in practice (see Dolado, Gonzalo, and Mayoral [2002] for details on the consequences of the truncation).

2.1 *The Sources of Fractional Integration in Inflation Data*

Before testing for the presence of the above-described features in inflation series, it would be enlightening to have some plausible explanations for their existence in the data.

Why can inflation be fractionally integrated? One plausible mechanism for generating long-run dependence in inflation could stem from the fact that some economically important shocks have long memory. Evidence of this behavior in geophysical and meteorological variables is well documented (see, among others, Mandelbrot and Wallis 1969). Some authors have argued that the prices of some goods (in particular, raw materials) could inherit this property which, in turn, they transmit to other related goods (see Haubrich and Lo 2001). It seems difficult, however, to assess the extent of this effect in a price index and, therefore, we will not pursue this explanation here.

A more satisfactory explanation of the FI behavior, however, is provided by models that produce strong dependence despite white noise shocks. By applying the aggregation results on heterogeneous agents, it is easy to show that FI could appear in inflation data. Let us consider a model of sticky prices as in Rotemberg (1987), where it is assumed that each firm faces a quadratic cost of changing its price.¹² It is well known that when this is the case, the dynamics of prices are given by

$$p_t^i = \vartheta p_{t-1}^i + (1 - \vartheta)p_t^{i*}, \quad (3)$$

where p and p^* represent the actual and optimal level of prices of firm i , and ϑ is a parameter that captures the extent to which imbalances are remedied in each period. Equation (3) can also be written as

$$\Delta p_t^i = \vartheta \Delta p_{t-1}^i + \nu_t^i, \quad (4)$$

¹²Quadratic costs of changing prices are equivalent, up to a first-order approximation, as far as aggregates are concerned, to a model such as Calvo (1983) where firms have a constant hazard of adjusting prices.

with $\nu_t^i = (1 - \vartheta)\Delta p_t^{i*}$. The parameter ϑ is a function of the adjustment costs and describes the speed of the adjustment, while $\vartheta/(1-\vartheta)$ is the expected time of adjustment. Since costs may differ across firms, it is natural to consider the case where ϑ may also depend on i . Then,

$$\Delta p_t^i = \vartheta^i \Delta p_{t-1}^i + \nu_t^i. \quad (5)$$

To build a price index, aggregation over a huge number of individual prices has to be considered (for instance, prices for the goods and services used to calculate the CPI are collected in eighty-seven urban areas throughout the United States and from about 23,000 retail and service establishments). Let us define the change in the price index Δp_t that verifies

$$\Delta p_t = \sum_{i=1}^N \Delta p_t^i.$$

Provided the distribution of ϑ^i verifies some (mild) semiparametric restrictions, Δp_t will display an FI behavior. Zaffaroni (2004) provides a full discussion of these restrictions. We will assume that ϑ belongs to a family \mathfrak{S} of continuous distributions on $[0,1)$ with density

$$\mathfrak{S}(\vartheta, d) \sim c\vartheta^{-d} \text{ as } \vartheta \rightarrow 0^+, \quad (6)$$

with $c \in (0, \infty)$. This is a very mild semiparametric specification of the cross-sectional distribution of ϑ . Zaffaroni (2004) shows that if ϑ is distributed according to (6), then the aggregated series will be $\text{FI}(d)$. The bigger the proportion of agents having values of ϑ^i close to one, the higher the memory of the process. In other words, if an important proportion of agents correct the imbalances between the actual and the optimal level of prices only by a very small amount each period, the inertia in the inflation rate will be very high since the main factor driving the dynamics will be past values of prices.

It is interesting to notice that the behavior of $\mathfrak{S}(\vartheta, d)$ within any interval $[0, \gamma]$ is completely unspecified. Many parametric specifications verify the restriction in (6), for instance, the uniform and the Beta distributions. Zaffaroni's results imply that if the value of the memory parameter d is known (or can be estimated), then it is possible to infer a precise indication of the shape of the cross-sectional

distribution of the ϑ'' s near one. This implies that it is possible to infer on certain aspects of the microenvironment using aggregate information only.

3. Evidence of FI Behavior in Inflation Data

In this section we analyze the evidence of FI behavior in inflation data through a series of steps. Subsection 3.1 reports the results of applying several estimation techniques that explicitly allow for FI. In order to obtain more robust results, both classical and Bayesian methods are employed. For all countries and across the different techniques, fractional values of d , distant from both $\{0, 1\}$, are found. Next, we perform different tests of integer versus fractional integration, and the results are reported in subsection 3.2. Finally, the possibility of having detected spurious long memory as a consequence of the existence of an unknown number of structural changes in the data has been analyzed in subsection 3.3.

3.1 Estimation Results

In order to obtain robust estimates of the parameters of interest, we have considered several of the most popular estimation techniques, namely, the Geweke and Porter-Hudak (1983) (GPH) semiparametric method and three parametric ones: exact maximum likelihood (EML; see Sowell 1992a), nonlinear least squares (NLS; Beran 1994), and a minimum distance estimator (MD; Mayoral 2004a).¹³ The estimated values of the memory parameter d are presented in table 3.

Several conclusions can be drawn from the inspection of this table. Firstly, the finding of fractional values of d , distant from the unit root, is robust across countries and across estimation methods. Most countries display values of d in the nonstationary ($d \geq 0.5$) but mean-reverting ($d < 1$) range, implying that, although very persistent, shocks are transitory. The semiparametric GPH method usually delivers slightly higher values of d than the other parametric techniques. This can be explained on the grounds that short-run

¹³NLS and EML have been computed with the ARFIMA package 1.0 for OX (Doornik and Ooms 2001), while MD has been implemented in MATLAB. Parametric models have been chosen according to the AIC information criteria.

Table 3. Estimation of FI(*d*) Models

	GPH	NLS	EML	MD
AU	0.78 (0.20)	0.79 (0.10)	0.69 (0.06)	0.74 (0.06)
AUS	0.78 (0.19)	0.69 (0.13)	0.80 (0.10)	0.73 (0.10)
BE	0.83 (0.21)	0.58 (0.10)	0.56 (0.06)	0.611 (0.08)
CA	0.76 (0.17)	0.69 (0.10)	0.73 (0.07)	0.69 (0.09)
DK	0.66 (0.16)	0.67 (0.11)	0.63 (0.07)	0.66 (0.07)
FI	0.74 (0.14)	0.59 (0.08)	0.60 (0.15)	0.62 (0.10)
FR	0.75 (0.21)	0.89 (0.21)	0.65 (0.06)	0.72 (0.08)
GE	0.94 (0.27)	0.58 (0.27)	0.61 (0.09)	0.68 (0.09)
GR	0.64 (0.30)	0.66 (0.10)	0.62 (0.05)	0.60 (0.06)
IT	1.19 (0.27)	0.72 (0.42)	0.66 (0.05)	0.69 (0.08)
JP	0.62 (0.09)	0.59 (0.16)	0.75 (0.10)	0.63 (0.10)
LX	0.74 (0.29)	0.69 (0.18)	0.68 (0.11)	0.65 (0.13)
NL	0.86 (0.20)	0.67 (0.14)	0.72 (0.12)	0.70 (0.11)
NZ	0.52 (0.41)	0.62 (0.14)	0.57 (0.08)	0.63 (0.10)
NO	0.64 (0.26)	0.66 (0.13)	0.55 (0.26)	0.64 (0.15)
PO	0.80 (0.22)	0.63 (0.10)	0.63 (0.07)	0.59 (0.10)
SP	0.90 (0.16)	0.61 (0.15)	0.60 (0.07)	0.65 (0.11)
SWE	0.58 (0.16)	0.59 (0.14)	0.52 (0.09)	0.59 (0.10)
SWI	0.56 (0.18)	0.62 (0.11)	0.59 (0.12)	0.61 (0.11)
UK	0.78 (0.20)	0.69 (0.22)	0.64 (0.10)	0.62 (0.10)
USA	0.66 (0.14)	0.68 (0.32)	0.72 (0.20)	0.69 (0.16)
Note: Standard deviation is shown in parentheses.				

correlation may bias the estimator upward (see Agiakloglou, Newbold, and Wohar 1992). The parametric methods present very similar values, and for most countries estimated values of d around 0.6–0.7 are found.

A problem often associated with parametric estimators of d is that they are very sensitive to the selection of the specific parametric model, so estimated values can vary greatly across different specifications. To overcome this problem, we have also computed some Bayesian estimates of d in order to take the model uncertainty into account. We follow Koop et al. (1997) and consider the sixteen possible combinations of ARFIMA models with $p, q \leq 3$. A uniform density for d in the interval $[0, 1.5]$ has been assumed. So, the method puts 2/3 of the prior mass on values of d implying nonpermanent shocks ($d < 1$) and 1/3 on values that correspond to permanent shocks ($d > 1$).

The outcome of the Bayesian estimation is reported in table 4. The mean and the standard deviation of d is provided for both the “best model” (the one with the highest posterior probability) and the “overall model,” which weights the sixteen ARFIMA models according to their posterior probabilities.¹⁴ Since the method computes the density function of d for each model, the probability that inflation is mean reverting ($P(d_i < 1)$) can be easily obtained and is also displayed in this table.

The results reported in table 4 suggest that there is a high variability associated with the estimation of d . In general, the Bayesian approach offers higher values of the memory parameter than the classical methods, although in almost all cases the estimated values remain below one. Moreover, the posterior probability of nonpermanent shocks ($d < 1$) is bigger than 2/3 (the a priori probability) for eighteen out of the twenty-one countries considered.

Summing up, the Bayesian analysis, in accordance with the classical approach, confirms the very persistent but mean-reverting behavior of inflation data.

¹⁴See Koop et al. (1997) for details on the estimation procedure. Computations have been carried out using the Fortran code provided by them.

Table 4. Bayesian Estimation of ARFIMA Models

	BEST ARFIMA		OVERALL ARFIMAS	
	Mean(d)	$P(d < 1/\text{data})$	Mean(d)	$P(d < 1/\text{data})$
AU	0.88 (0.19)	0.75	0.82 (0.20)	0.82
AUS	0.34 (0.06)	1	0.34 (0.06)	1
BE	0.86 (0.14)	0.90	0.87 (0.15)	0.76
CA	0.99 (0.26)	0.55	0.85 (0.21)	0.74
DK	0.85 (0.21)	0.71	0.87 (0.23)	0.63
FI	0.62 (0.06)	1	0.67 (0.15)	0.95
FR	0.66 (0.07)	1	0.68 (0.14)	0.93
GE	0.78 (0.33)	0.86	0.83 (0.26)	0.76
GR	0.64 (0.06)	1	0.78 (0.17)	0.82
IT	0.73 (0.18)	0.92	0.66 (0.13)	0.96
JP	0.64 (0.10)	0.99	0.62 (0.21)	0.91
LX	0.98 (0.31)	0.65	0.83 (0.22)	0.78
NL	0.91 (0.28)	0.54	0.79 (0.25)	0.76
NZ	0.91 (0.31)	0.60	0.85 (0.22)	0.66
NO	0.57 (0.06)	1	0.71 (0.19)	0.86
PO	1.33 (0.12)	0.03	1.14 (0.18)	0.25
SP	1.30 (0.30)	0.30	1.07 (0.31)	0.52
SWE	0.42 (0.05)	1	0.80 (0.24)	0.74
SWI	0.60 (0.06)	1	0.65 (0.17)	0.94
UK	0.60 (0.06)	1	0.80 (0.15)	0.75
USA	0.58 (0.19)	0.97	0.64 (0.22)	0.86
Note: Standard deviation in shown in parentheses.				

3.2 Testing Fractional versus Integer Integration

Tables 3 and 4 support our initial hypothesis of the fractionally integrated behavior of inflation data and that the order of integration is, in general, far from both zero and one. But one could argue that this could be the case even if the series has an integer degree of integration since it would be very unlikely to obtain an exact integer value for d . In this section, we will formally test these hypotheses.

Several authors have found evidence in favor of the existence of a unit root in inflation (see, for instance, Pivetta and Reis 2004). Other authors, such as Cogley and Sargent (2001), postulate an $I(0)$ representation for inflation on the basis that nonstationary ones are not plausible since they would imply an infinite asymptotic variance of inflation. They argue that this could never be optimal if the central bank's loss function includes the aforementioned variance. We will show below that when the possibility of fractional integration is considered, both the $I(0)$ and the $I(1)$ representations are rejected in our data set.

The simplest test is to build confidence intervals around the estimated values of d reported in table 3. Although simple, this approach has an important drawback: usually intervals are too wide and most hypotheses cannot be rejected (see Sowell 1992a). Fortunately, other simple and more powerful methods are available in the literature. To test the unit root versus the FI hypothesis, the Fractional Dickey-Fuller (FDF) test (see Dolado, Gonzalo, and Mayoral 2002, 2003) has been employed. This test generalizes the traditional Dickey-Fuller test of $I(1)$ against $I(0)$ to the more general framework of $I(1)$ versus $FI(d)$. It is based upon the t-ratio associated with the coefficient of $(1-L)^d y_{t-1}$ in a regression of $(1-L)y_t$ on $(1-L)^d y_{t-1}$ and, possibly, some lags of $(1-L)y_t$ to account for the short-run autocorrelation of the process and/or some deterministic components if the series displays a trending behavior or initial conditions different from zero.¹⁵ Table 5 presents the results of applying the FDF test to

¹⁵The FDF invariant regression that has been run is equal to $\Delta y_t = \alpha_1 \tau_{t-1}(d) + \phi \Delta^d y_{t-1} + \sum_{j=1}^k \psi_j \Delta y_{t-j} + a_t$, and a number of lags of Δy_t equal to two was chosen according to the BIC criterion. The coefficient α_1 is associated to the deterministic components (a constant; see Dolado, Gonzalo, and Mayoral 2003). The term $\tau_t(d)$ is defined as $\tau_t(d) = \sum_{i=0}^{t-1} \pi_i(d)$, where the coefficients $\pi_i(d)$ come from the expansion of $(1-L)^d$ as defined in equation (2).

Table 5. FDF Test (I(1) versus FI(d)). $H_0 : d_0 = 1$; $H_1 : d = d_1$

$H_1 :$	$d_1 = 0.6$	$d_1 = 0.7$	$d_1 = 0.8$	$d_1 = 0.9$
AU	-8.76**	-4.65**	-4.68**	-4.69**
AUS	-8.56**	-8.54**	-8.47**	-8.36**
BE	-7.39**	-7.53**	-7.62**	-7.69**
CA	-5.92**	-5.66**	-3.73**	-3.70**
DK	-6.14**	-6.05**	-5.94**	-5.81**
FI	-5.45**	-5.19**	-4.90**	-3.20**
FR	-4.34**	-4.12**	-3.27**	-3.26**
GE	-6.77**	-6.79**	-6.77**	-6.72**
GR	-5.79**	-5.62**	-5.43**	-5.24**
IT	-4.82**	-2.87**	0.01	0.17
JP	-8.73**	-4.52**	-4.51**	-4.50**
LX	-7.32**	-4.55**	-4.60**	-4.65**
NL	-6.86**	-6.68**	-6.49**	-5.89**
NZ	-9.31**	-4.70**	-4.56**	-4.41**
NO	-6.77**	-6.50**	-6.22**	-3.12**
PO	-8.04**	-4.40**	-4.31**	-4.20**
SP	-7.88**	-7.65**	-3.80**	-3.89**
SWE	-6.07**	-6.03**	-5.79**	-5.78**
SWI	-5.86**	-5.58**	-3.73**	-3.68**
UK	-6.07**	-5.84**	-5.58**	-5.32**
USA	-2.27*	-2.18*	-2.11*	-2.04*
Note: *,** denote rejection at the 5 percent and the 1 percent level, respectively. Critical values: $N(0, 1)$.				

this data set. Several alternative hypotheses have been considered ($d = 0.6, 0.7, 0.8$, and 0.9). The conclusion of this table is clear: the unit-root model is clearly rejected (usually at the 1 percent significance level) against fractionally integrated alternatives in all countries.

Next, we test for FI versus short memory ($I(0)$). To this end, a point-optimal test recently proposed by Mayoral (2004b) has been implemented and the results are presented in table 6. The test works as follows. Given the characteristics of the inflation data, the following DGP has been considered:

$$\begin{aligned} y_t &= \mu + x_t \\ \Delta^{d_i} x_t &= u_t, i = \{0, 1\}, \end{aligned}$$

where μ is a constant, u_t is a linear $I(0)$ process, and $d = d_0$ and $d_1 = 0$ are, respectively, the integration orders under H_0 and H_1 . Under the Neyman-Pearson lemma, the most powerful test will reject the null hypothesis of $d = d_0$ for small values of $L(d, \sigma)|_{H_1} - L(d, \sigma)|_{H_0}$, where L is the log-likelihood function. After some manipulation, the critical region of the most powerful test for these hypotheses is given by

$$\frac{\sum (y_t - \mu)^2}{\sum (\Delta^{d_0} (y_t - \mu))^2} < k_T. \quad (7)$$

The asymptotic distribution of this statistic (scaled by T^{1-2d}) is not standard, and critical values can be found in Mayoral (2004b) for the case where u_t is i.i.d. When u_t is a general linear short-memory process, a nonparametric correction should be introduced using any of the standard techniques available in the literature (see Mayoral 2004b).

To interpret the figures reported in table 6, it is important to notice that the test is consistent (rejects the null hypothesis of $FI(d_0)$ for large T) if the true integration order, d^* , is smaller than the integration order used as the null hypothesis, d_0 . Consequently, whenever $d_0 > d^*$, the test will reject the $FI(d_0)$ hypothesis. For example, if the true integration order is $d^* = 0.7$ but $d_0 = 0.9$ is taken as H_0 , the test will tend to reject the hypothesis of $d_0 = 0.9$.

The results in table 6 are very homogeneous across countries. For moderate values of d , around 0.6–0.7 and even 0.8 for most countries, the null hypothesis of FI cannot be rejected. Nevertheless, for higher values of d_0 ($d_0 = 0.9$), the same null is rejected. This result confirms the outcome of the estimation methods in table 3 since, according to this table, the true integration orders are around 0.7. Therefore, taking into account the properties of the test, when higher d_0 's are

Table 6. Test of $FI(d)$ versus $I(0)$

$H_0 :$	R ^c Test (Mayoral 2004)			
	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$
AU	1.136	0.456	0.175	0.064*
AUS	0.592	0.257	0.071*	0.024*
BE	0.550	0.196*	0.069*	0.024*
CA	1.315	0.547	0.217	0.083*
DK	0.899	0.339	0.124	0.044*
FI	1.054	0.438	0.174	0.067*
FR	0.939	0.397	0.162	0.064*
GE	0.839	0.327	0.123	0.044*
GR	0.737	0.273	0.098*	0.035*
IT	1.434	0.614	0.251	0.099
JP	1.013	0.408	0.158	0.059*
LX	1.125	0.466	0.184	0.070*
NL	0.513	0.282	0.063*	0.022*
NZ	0.817	0.314	0.117*	0.042*
NO	1.006	0.400	0.154	0.057*
PO	1.218	0.483	0.184	0.068*
SP	1.079	0.448	0.178	0.068*
SWE	1.019	0.405	0.155	0.058*
SWI	0.840	0.347	0.138	0.053*
UK	1.014	0.412	0.161	0.061*
USA	1.225	0.535	0.225	0.091*
Crit. Values (5% <i>S.L.</i>)	0.502	0.241	0.122	0.092

employed, the test should reject $H_0 : d = d_0$, as it actually does. Thus, the test supports the hypothesis of FI behavior with a degree of integration close to 0.7.

3.3 Testing Fractional Integration versus Structural Breaks

It is well known that it is very difficult to provide an unambiguous answer as to whether a process is fractionally integrated or is short memory plus some deterministic components perturbed by sudden changes. Several authors have pointed out that many standard

techniques for detecting persistence can spuriously find this property in short-memory processes when there is parameter instability (e.g., Bhattacharya, Gupta, and Waymire [1983], Künsh [1986], Perron [1989], Teverosky and Taqqu [1997], Giraitis, Kokoszka, and Leipus [2001], Mikosch and Starica [2004], Perron and Zhu [2005], and many others). Other authors have studied the opposite effect, that is, how conventional procedures for detecting and dating structural changes tend to find spurious breaks, usually in the middle of the sample, when in fact there is only fractional integration (see Nunes, Kuan, and Newbold [1995], Krämer and Sibbertsen [2002], and Hsu [2000]). Therefore, although there is a general consensus on the fact that most economic series are nonstationary, it is often difficult to be sure about the source of the nonstationarity, that is, whether it comes from a high degree of persistence or from the existence of parameter changes.

In view of these results, it is not surprising that evidence supporting both the existence of breaks in the mean (section 1) and strong persistence (subsections 3.1 and 3.2) is found for the same data set. For the purposes of this article, distinguishing between these two models is crucial since they have very different implications in terms of the degree of persistence. Thus, we now explore the possibility that the existence of different regimes in the mean in an otherwise short-memory process could be generating spurious memory in the inflation rate. To do so, an extension of the test described in subsection 3.2 has been employed. The aim of the test is to determine if the persistence observed in the data is real or is an artifact of other phenomena such as the existence of breaks. More specifically, the hypotheses of $FI(d_0)$ versus $I(0)$ with a break in the level are considered. The test works as follows: let T_B be the time when the break occurs and $\omega = T_B/T$ the parameter that describes the location of the break point in the sample. To allow for breaks in the level, the dummy variable $DC_t(\omega) = 1$ if $t > T_B$ and 0 otherwise is defined. Since the date where the break occurs is unknown, the test has a critical region given by

$$\min_{\omega} \frac{\min_{\alpha_1, \alpha_2} \sum (y_t - \alpha_1 - (\alpha_2 - \alpha_1)DC_t(\omega))^2}{\min_{\alpha_0} \sum (\Delta^{d_0}(y_t - \alpha_0))^2} \leq k_T, \quad (8)$$

where the minimization is carried out in $\omega \in \Omega$, where, following Andrews (1993), $\Omega = [0.15, 0.85]$. The distribution of the statistic in

Table 7. Tests of $FI(d)$ versus Breaks

$H_0 :$	0.6	0.7	0.8	0.9
AU	0.4284	0.1132*	0.0304*	0.0084*
AUS	0.1997*	0.0602*	0.0184*	0.0051*
BE	0.4121	0.0817*	0.0091*	0.0021*
CA	0.9027	0.2466	0.0678*	0.0181*
DK	0.1953*	0.0539*	0.0152*	0.0043*
FI	0.8050	0.2284	0.0645*	0.0181*
FR	0.8796	0.3228	0.1309	0.0400*
GE	0.4001	0.0979*	0.0188*	0.0053*
GR	0.7318	0.2077	0.0534*	0.0032*
IT	1.7063	0.4857	0.1469	0.0401
JP	0.3638*	0.0987*	0.0269*	0.0070*
LX	0.6286	0.1688*	0.0456*	0.0121*
NL	0.1350*	0.0299*	0.0086*	0.0025*
NZ	0.3422*	0.0998*	0.0224*	0.0062*
NO	0.6561	0.1847	0.0520*	0.0141*
PO	0.5419	0.1476*	0.0409*	0.0114*
SP	0.6018	0.1658*	0.0460*	0.0129*
SWE	0.3004*	0.0823*	0.0229*	0.0061*
SWI	0.6391	0.1782*	0.0494*	0.0131*
UK	0.8408	0.2339	0.0648*	0.0180*
USA	1.4462	0.4098	0.1136	0.0313*
Crit. Values (5% <i>S.L.</i>)	0.399	0.175	0.0844	0.0404

(8), scaled by T^{2d-1} , is nonstandard, and critical values are provided in Mayoral (2004b). Again, since short-term structure is allowed, the test statistic has been corrected using standard nonparametric techniques (see Mayoral [2004b] for details).

Table 7 summarizes the output of the tests. For fifteen out of the twenty-one countries considered, the null hypothesis of fractional integration cannot be rejected.¹⁶ The countries for which this

¹⁶Notice that the simulations reported in Mayoral (2004b) show that the employed techniques are very powerful against a wide variety of DGPs under the alternative hypothesis, with rejection rates ranging from 90 to 100 percent for this sample size. Then, we are confident that the nonrejection of the null hypothesis is not due to lack of power.

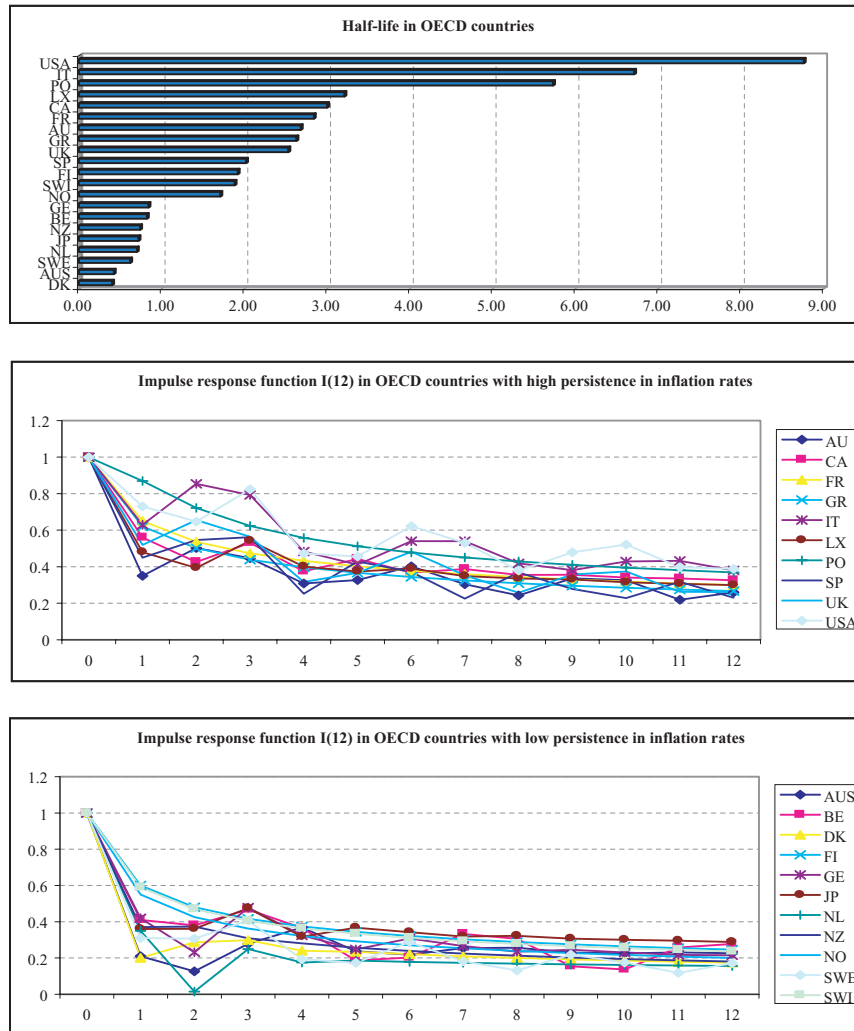
hypothesis is dismissed are Austria, Denmark, Japan, Netherlands, New Zealand, and Sweden. Two more countries, Belgium and Germany, are on the border between rejection and nonrejection. For these eight countries, the hypothesis of $d > 0.5$ versus $I(0)+$ breaks has also been tested, and the null was only rejected for four of them (NL, DK, AUS, and SWE). To understand this finding, it is interesting to look at the first graph of figure 2, which depicts the half-life measure of persistence, and figure 3, which shows the $IRF(h)$ of these countries. Notice that, in figure 2, the latter four countries appear at the very bottom of the graph, implying that they are the least persistent. Right above those four, JP, NZ, BE, and GE are found. Therefore, it seems that at least some of the persistence that has been found in these series is spurious and derives from the existence of some breaks in the average level of inflation.

4. Measuring Persistence

In sections 2 and 3 we have presented an economic explanation and some robust empirical evidence supporting the hypothesis of fractionally integrated behavior in inflation data. Bearing in mind these results, we turn now to the main goal of the paper, the measurement of inflation persistence. In the following, by *persistence* we mean the long-term effect of a shock to the series.

In this section we provide various persistence measures that permit an adequate comparison of inflation inertia across countries and their evolution over time. The relevance of explicitly considering FI alternatives will become clear now. Our results demonstrate that, although in the short run the estimated persistence from the ARIMA and ARFIMA specifications is similar, the medium- and long-run implications are very different. This is due to the fact that, in order to model nonstationarity, ARIMA models necessarily impose the restriction of permanent shocks, while the more flexible ARFIMA formulations are able to characterize nonstationarity without imposing such a restriction. We show that some scalar measures of persistence, such as the sum of the AR coefficients (or its equivalent, the *cumulative impulse response* [see Andrews and Chen 1994]) are not suitable for measuring persistence in this context since they deliver exactly the same value for all $FI(d)$ processes with $d > 0$ (equal to one for the former and to ∞ for the latter), despite the fact that processes

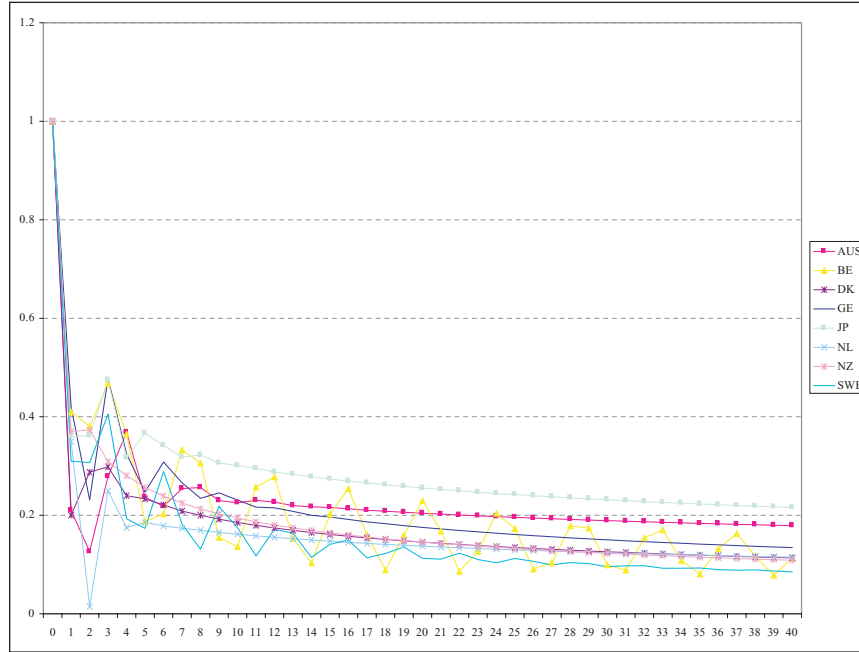
Figure 2. Half-Life and Impulse Response Functions in the Middle Run



in this group are of a very different character. In relation to this behavior, we also discuss some potential pitfalls that these techniques may present when used in applied work.

There are several ways to measure persistence, each with its virtues and faults. In the next subsection, we describe the tools that

Figure 3. Impulse Response Functions of Countries with Inflation Process $I(0)$ with a Break in the Mean



will be used in this analysis. In order to have an accurate picture of this important property, we consider the estimation under both the classical and the Bayesian approach. Subsections 4.2 and 4.3 report the corresponding results.

4.1 Measuring Persistence with FI Processes

We consider three different tools in order to evaluate persistence—firstly, the *impulse response function (IRF)*, which measures “the effect of a change in the innovation ε_t by a unit quantity on the current and subsequent values of y_t ” (see Andrews and Chen 1994, 189). This measure is problematic because it is a vector, not a scalar, and, therefore, could be more difficult to interpret. For this reason, we also consider two scalar measures that will be described below.

For stationary series, the impulse responses are the coefficients of their Wold decomposition. For $I(1)$ processes, the $IRF(h)$ is usually

computed¹⁷ as the sum from zero to h of the impulse response coefficients of the first differences of the original series.¹⁸ The above-mentioned expressions are embedded in the general formulation of the $IRF(h)$ of an ARFIMA(p, d, q) process. This is defined as the h -th coefficient of $A(L) = (1 - L)^{-d}\Phi(L)^{-1}\Theta(L)$, where $\Phi(L)$ and $\Theta(L)$ are the AR and MA polynomials, respectively. The corresponding coefficients can be computed according to the following formula (see Koop et al. [1997] for details):

$$IRF(h) = \sum_{i=0}^h \pi_i(-d)J(h-i), \quad (9)$$

where each $\pi_i(-d)$ comes from the binomial expansion of $(1 - L)^{-d}$ and is defined in (2), and $J(\cdot)$ is the standard ARMA(p, q) impulse response, given by

$$J(i) = \sum_{j=0}^q \theta_j f_{i+1-j},$$

with $\theta_0 = 1$, $f_h = 0$ for $h \leq 0$, $f_1 = 1$, and

$$f_h = -(\phi_1 f_{h-1} + \dots + \phi_p f_{h-p}), \quad \text{for } h \geq 2.$$

Notice that if $d = 1$, $\pi_i(-1) = 1$ for all i and, therefore, the traditional IRF for I(1) processes is recovered, i.e., $IRF(h) = \sum_{i=0}^h J(h-i)$ (see Campbell and Mankiw 1987, 861). The limit behavior of the $IRF(h)$ when $h \rightarrow \infty$ depends upon the value of d and verifies

$$IRF(\infty) = \begin{cases} 0, & \text{if } d < 1, \\ \Phi(1)^{-1}\Theta(1), & \text{if } d = 1, \\ \infty & \text{if } d > 1. \end{cases} \quad (10)$$

¹⁷A different approach, which will not be pursued in this article, is to compute impulse responses based on estimating local projections at each period of interest (see Jordà 2005).

¹⁸Since the $IRF(h)$ function in the I(1) case is computed by accumulating the individual I(0) impulse responses, it is often called *cumulative impulse response function* (see, for instance, Diebold and Rudebusch 1989). However, we will not use this terminology here in order to avoid confusion with other measures that share a similar name. This is the case of the *cumulative impulse response* (see Andrews and Chen 1994).

Expression (10) means that the effect of a shock is transitory for $d < 1$, as the long-term impact of any shock is equal to zero. By contrast, shocks are permanent for any $d \geq 1$. If the process contains a unit root ($d = 1$), the long-run effect of the shock is bounded away from zero and finite and is given by the sum of the Wold coefficients of its stationary transformation (or alternatively, by $\Phi(1)^{-1}\Theta(1)$ if it admits an ARMA representation). Finally, for any $d > 1$ the effect of any shock is magnified and the final impact is not bounded. Based on this behavior, Hauser, Pötscher, and Reschenhofer (1999) have criticized the use of ARFIMA models for measuring persistence. They argue that, although the ARIMA class is nested within the more general ARFIMA formulation, it would not be wise to use these models if the true DGP is in fact ARIMA. This is so because if $d = 1$, it would be extremely unlikely to obtain exactly this estimated value in finite samples. Thus, since the $\text{IRF}(\infty)$ is highly discontinuous, this would be equivalent to imposing an a priori value of this function either equal to zero (if $\hat{d} < 1$) or to infinity (if $\hat{d} > 1$). According to their view, imposing these long-term restrictions would also adversely affect the estimation of the $\text{IRF}(h)$ for finite values of h (see the simulations provided in Hauser, Pötscher, and Reschenhofer (1999, Table 1).

We agree with them that, for the purpose of persistence estimation, it is important to treat the ARFIMA and the ARIMA classes as two different groups of models, despite the fact that one contains the other. This is one of the reasons that led us to apply an ample battery of tests to distinguish between these formulations in our data set. But, in our opinion, it does not follow from here that the use of ARFIMA processes is inadequate to measure persistence. There are several ways in which the criticisms in Hauser, Pötscher, and Reschenhofer (1999) can be answered. The most obvious is that their misspecification argument can be easily reversed, that is, if the DGP is $\text{FI}(d)$ but an ARIMA model (with integer d) is fitted to the data to compute the impulses, the (wrong) long-term restrictions imposed by the ARIMA specification might bias the estimates as a result of the misspecification. Since the empirical evidence found in the previous section supports the better fit of the ARFIMA over the ARIMA model, the use of the former is well justified. The estimated values of d obtained for our data set are, in general, less than one, which means that the $\text{IRF}(\infty)$ associated with these processes is

zero. This restriction reflects the main finding of section 3: the inflation rate is best characterized as a nonstationary but mean-reverting process. If this condition is true, imposing a unit root to compute the impulses will result in higher estimated persistence, since the permanent shock restriction will upwardly bias the estimates. This fact is illustrated in table 9, shown in subsection 4.3.

Finally, we are aware that it is not possible to be certain about the true nature of the DGP. So, in order to avoid possible biases in our estimates stemming from imposing a possibly incorrect long-term restriction, in subsection 4.3 we estimate the impulse responses using a Bayesian approach that explicitly acknowledges model uncertainty. By allowing for a strictly positive probability mass on the $I(1)$ model, we will be able to obtain a continuous impulse response function with a strictly positive and bounded value at infinity. To do so, we will follow the approach of Koop et al. (1997).

In addition to the IRF, two scalar measures of persistence are also reported: the half life (HL), defined as the number of periods that a shock needs to vanish by 50 percent, and ρ_{40} , which is given by

$$\rho_{40} = 1 - 1 / \sum_{h=0}^{40} IRF(h).$$

This quantity can be interpreted as a truncated version of the sum of the AR coefficients (see Andrews and Chen 1994), defined as

$$\rho(1) = 1 - 1 / \sum_{h=0}^{\infty} IRF(h),$$

and is introduced here in order to overcome the problems that this measure presents in this context. It turns out that $\rho(1) = 1$ for any integrated process with an integration order strictly greater than zero. This is so because any invertible $FI(d)$ process admits an $AR(\infty)$ representation, given by

$$(1 - L)^d C(L)^{-1} y_t = \varepsilon_t,$$

where the innovations $\{\varepsilon_t\}_{-\infty}^{\infty}$ are white noise and $C(L)$ is the polynomial of the Wold representation of the $I(0)$ variable $(1 - L)^d y_t$. For any $d > 0$, $L = 1$ is a root of the polynomial $(1 - L)^d C(L)^{-1}$. Calling

$\Lambda(L) = (1 - L)^d C(L)^{-1} = 1 - \sum_{i=1}^{\infty} \lambda_i L^i$ and noticing that $L = 1$ is a root of $\Lambda(L)$, it follows that $1 - \sum_{i=1}^{\infty} \lambda_i 1^i = 0$, which implies that $\rho(1) = \sum_{i=1}^{\infty} \lambda_i = 1$. An equivalent way of looking at this result is by considering the *cumulative impulse response* (CIR), given by

$$CIR = 1/(1 - \rho(1)) = \sum_{i=0}^{\infty} IRF(h).$$

This measure is proportional to the spectral density at frequency zero (see Andrews and Chen 1994). Since the spectral density of any FI(d) process with $d > 0$ is unbounded at frequency zero, it follows that $CIR = \infty$ for any FI(d) process with $d > 0$. Since the degree of persistence varies a great deal across the different values of d in this range, it follows that $\rho(1)$ cannot be taken as a good measure of persistence in this case. To overcome this problem, we consider a truncated version of it, ρ_{40} , which, instead of considering the sum of the IRF(h) for $h = 1, \dots, \infty$, only considers the first forty coefficients (which we identify with the long run). Interestingly, this measure can be considerably far from 1 for moderate values of d (for instance, in an FI(i) process with $i = \{0.1, 0.2, 0.3\}$, it would be around 0.35, 0.59, and 0.74, respectively).

4.2 Classical Estimation

We now report the estimated values of the three tools presented above, obtained using classical techniques. Table 8 presents the IRF(h) at different time horizons h , namely, $h = 4, 12$, and 40, representing the short, middle, and long run, respectively. In addition, columns 4 and 5 report the values of the HL and ρ_{40} , respectively. The full path of IRF(h) for different time horizons is displayed in figure 4.

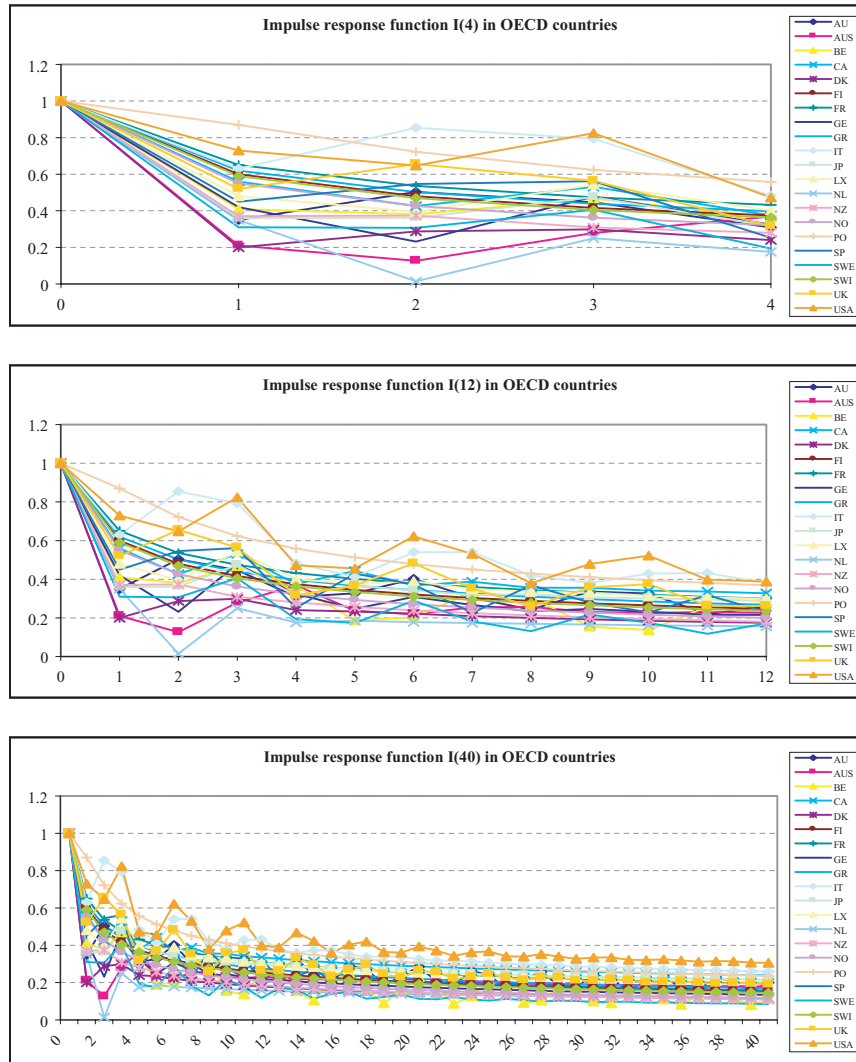
The information in table 8 can be summarized as follows. For the twenty-one industrial countries, the IRF decreases in the middle- and long-run horizons, although the remaining effect of shocks differs considerably across countries, ranging from 38 percent for the United States versus 17 percent for Sweden in the middle-run horizon, and 30 percent versus 8 percent for the same countries in the long term. The ρ_{40} measure oscillates within the interval $[0.90, 0.96]$, confirming the high persistence of the series. It is interesting to compare this result with the one obtained in Pivetta and Reis (2004).

Table 8. IRF and Scalar Measures of Persistence

	IRF(4)	IRF(12)	IRF(40)	HL	ρ_{40}
AU	0.3087	0.2601	0.2002	2.68	0.94
AUS	0.3684	0.2266	0.1789	0.42	0.94
BE	0.3650	0.2779	0.1135	0.82	0.92
CA	0.3749	0.3267	0.2386	3.00	0.95
DK	0.2399	0.1742	0.1134	0.40	0.91
FI	0.3744	0.2461	0.1531	1.92	0.90
FR	0.4324	0.3000	0.1980	2.84	0.92
GE	0.3246	0.2152	0.1344	0.84	0.93
GR	0.3969	0.2666	0.1698	2.63	0.94
IT	0.4829	0.3805	0.2585	6.71	0.96
JP	0.3168	0.2879	0.2158	0.72	0.95
LX	0.4101	0.2991	0.2036	3.21	0.95
NL	0.1751	0.1553	0.1149	0.70	0.91
NZ	0.2804	0.1803	0.1087	0.74	0.91
NO	0.3316	0.2002	0.1173	1.71	0.92
PO	0.5581	0.3689	0.2360	5.73	0.96
SP	0.2521	0.2307	0.1573	2.02	0.91
SWE	0.1930	0.1713	0.0853	0.62	0.90
SWI	0.3634	0.2363	0.1453	1.88	0.93
UK	0.3166	0.2615	0.1902	2.53	0.95
USA	0.4726	0.3883	0.3058	8.76	0.96
Notes: IRF(h), $h = 4, 12, 40$ denote the impulse response function. HL is the half life defined as the number of periods that a shock needs to vanish by 50 percent. ρ_{40} is computed as $1 - 1 / \sum_{h=1}^{40} \text{IRF}(h)$.					

They estimate $\rho(1)$ for the U.S. inflation rate from an AR(p) specification, where $p = 3$ is chosen according to the Bayesian information criterion (BIC). They obtain estimates of this quantity around 0.95 and they conclude that inflation has a unit root and, therefore, that shocks to inflation are permanent. Nevertheless, as has been shown above, a value of $\rho(1)$ close to one does not imply an integer unit root but only a fractional one. Thus, one cannot say much about inflation

Figure 4. Impulse Response Functions



persistence just by looking at this quantity since very different types of integrated processes share this property.

In order to illustrate this, we have carried out a small Monte Carlo experiment: we have generated 5,000 ARFIMA(0, d , 0) processes with a value of $d = 0.7$ (which is approximately the estimated

value for U.S. inflation obtained in section 3; see table 3). Then we have fitted an AR(p) process using the BIC as in Pivetta and Reis (2004). Although the DGP is AR(∞), any sensitive information criteria will select a much shorter lag length. In fact, we have found that, on average, the chosen lag length is $p = 3$ and the mean (median) of $\rho(1)$ is 0.89 (0.90) with a standard deviation equal to 0.26. This example shows that the traditional interpretation that identifies $\rho(1) \approx 1$ with the existence of an integer unit root is clearly unfounded and could lead to persistence overestimation if one concludes from here that shocks are permanent.

A related problem can be found in Cogley and Sargent (2001). These authors assume that inflation is stationary. In order to impose this assumption, they truncate the parameter space so that the largest autoregressive root (LAR) is strictly less than one. Thus, they are imposing not only stationarity (which is compatible with an LAR equal to one in a fractional model with $d < 0.5$) but short memory (bounded spectral density). As Pivetta and Reis (2004) point out, this truncation could strongly bias the results toward lower values of persistence.

Figure 2, shown previously in subsection 3.3, ranks the different countries in accordance with their HL value and shows that its behavior varies a lot across them. Broadly speaking, two groups can be distinguished: the *low inflation persistence* group, exhibiting an HL of less than two periods (equivalent to six months), and the *high inflation persistence* group, with an HL superior to two periods. In the first group, the Scandinavian countries SWE, FI, and NO, together with JP, NZ, and SWI, can be found. All of them show a low inflation rate in most of the period with a mean around 4 percent. Other countries such as AUS, DK, NL, BE, and GE are also included in this group and are characterized by a tight monetary discipline and an implicit commitment with the German currency, whether they belonged to the European Monetary System or not. The members of the second group are AU, CA, FR, GR, LX, IT, PO, SP, UK, and USA with an inflation mean around 6 percent. The United States is the country with the highest HL, with a value around two years. However, this quantity is considerably smaller than that obtained by Pivetta and Reis (2004), who present figures of the HL of more than five years. This important difference in magnitude is a consequence of the use of the I(1) (permanent shocks) specification instead of

the $FI(d)$ one with $d < 1$ (mean-reverting shocks) employed in this article.

4.3 *Bayesian Estimation*

We now turn to the Bayesian estimation of inflation persistence. Although we have found abundant evidence against integer values of d , in this subsection we acknowledge our uncertainty by considering different combinations of ARIMA and ARFIMA models. The main motivation for undertaking this analysis is to overcome the criticism presented by Hauser, Pötscher, and Reschenhofer (1999). They argued that ARFIMA models may not be appropriate for measuring persistence because they imply a limit behavior of $IRF(\infty)$ which is either zero (if $d < 1$) or infinity (if $d > 1$). Nevertheless, using Bayesian techniques, it is possible to achieve a continuous distribution of the $IRF(\infty)$ in the interval $[0, \infty)$ if a strictly positive prior probability is assumed for the integer values of d . Following Koop et al. (1997), we have considered sixteen ARIMA models (where $d = 1$ is imposed) and sixteen ARFIMA models, corresponding to the different combinations of ARMA parameters, with $p, q \leq 3$ in both cases. In order to determine the prior probabilities assigned to both groups of models, we will use the posterior probabilities of $d_i < 1$ that were obtained in subsection 3.1. It is clear that $P(d \geq 1) = 1 - P(d < 1)$ and, therefore, we can use this expression as an upper bound for the probability of $P(d = 1)$. This quantity will be used as the prior probability for the ARIMA models. Table 9 reports the IRF evaluated at different time horizons for the best ARIMA and ARFIMA models (the ones with highest posterior probability) and also for the OVERALL model, constructed as a sum of the thirty-two models weighted by their posterior probabilities.

Bayesian IRFs present slightly higher values than those obtained under the classical paradigm, but in general, the nonpermanent character of shocks and the classification among countries is maintained. It is also interesting to compare the results obtained from the ARFIMA and ARIMA models. Both deliver very similar values in the short run, but they are very different in the medium and long run. Therefore, if only ARIMA alternatives are considered, it is very easy to conclude that shocks are much more persistent than they actually are.

[illegible]

Summarizing, in agreement with previous findings, this section confirms the high degree of inflation inertia. The United States emerges as the country with the highest inflation persistence in contrast to the Nordic countries, which display the lowest rates. Interestingly, high inertia is compatible with mean-reverting shocks in the framework considered in this article, a feature that cannot be captured in the $I(1)$ setup. This finding is relevant in many contexts, for instance, if one is interested in testing monetary neutrality.

5. Changes in Persistence

Another issue that has been widely studied recently is the stability of persistence over time. Changes in persistence may have a decisive impact on monetary strategy design. Some authors have pointed out that, if there is a decrease in inflation persistence, tests of the natural rate hypothesis in the spirit of Solow (1968) or Tobin (1968) may reject the null hypothesis of monetary neutrality as a consequence of this decrease. On the other hand, monetary policy is usually implemented in a more aggressive way in a context where inflation persistence increases. Furthermore, many macroeconomic models incorporate a measure of the persistence of inflation and, if persistence is not constant over time, Lucas's critique could apply.

The hypothesis of the stability of inflation persistence has been tested recently in various articles. Nevertheless, no consensus seems to have been reached. On the one hand, authors such as Taylor (2000), Cogley and Sargent (2001), and Kim, Nelson, and Piger (2004) have found that inflation inertia has decreased in recent years as a result of a general deflationist process, the implementation of target rules, and a more credible performance of central banks.¹⁹ On the other hand, Stock (2001), Batini (2002), Levin and Piger (2003), O'Reilly and Whelan (2004), Hondroyiannis and Lazaretou (2004), and Pivetta and Reis (2004) have found little evidence of changes in persistence for different countries.

In many of the latter papers, the decrease in persistence has been tested by checking whether the sum of the AR coefficients has

¹⁹By using a more historical perspective, some authors have found changes in persistence linked to different monetary regimes (c.f. Barsky [1987], Alogoskoufis and Smith [1991], Alogoskoufis [1992], Bordo and Schwartz [1999], Kim [2000], and Benati [2002]).

changed from one to a value strictly smaller than one. But, as was pointed out in section 4, this procedure is not completely correct if FI is allowed for. If a process is FI(d), the sum of the AR coefficients is equal to one for any $d > 0$. So, a decrease in persistence, associated with a lower value of d , does not have any theoretical impact on this sum (whose value will remain equal to one) as long as the new d is larger than zero. Therefore, a test based on the aforementioned criteria is likely to have very low power.

In this section, we will explore the stability of inflation persistence using a different approach. We will directly test whether the memory parameter d has remained constant over time or not. In order to do so, a Lagrange multiplier (LM) test of the stability of d will be applied (see Mayoral [2005] for further details). The following DGP is considered:

$$\begin{aligned} y_t &= \mu + x_t \\ \Delta^{d+\theta D_t(\omega)} x_t &= \Phi(L)^{-1} \Theta(L) \varepsilon_t. \end{aligned}$$

The process y_t is the sum of a constant term, μ , and a fractionally integrated process x_t . The parameter $\omega = t_0/T$ describes the location of a change in the value of d in the sample that, if it occurs, happens at time t_0 . $D_t(\omega)$ is a dummy variable that takes the value one if $\omega T < t$ and zero otherwise. The process ε_t is assumed to be i.i.d. and $\Phi(L)$, $\Theta(L)$ are the standard AR and MA polynomials, respectively. Under the null hypothesis, there is no change in persistence and, therefore, $\theta = 0$. Under H_1 , a single break in d is allowed to take place so that θ can take both positive and negative values, indicating an increase or a decrease of persistence, respectively. The test is developed following Andrews (1993) and works as follows: assuming normality, the test statistic derived under the LM principle for any fixed ω is given by

$$LM_T(\omega) = S_T(\omega)' A^{-1} S_T(\omega),$$

where S_T is the score obtained by deriving the likelihood function with respect to θ ,

$$S_T(\omega) = \frac{\partial L(d, \theta, \sigma^2, \beta, \omega)}{\partial \theta} = \omega T \sum_{i=1}^{\omega T} \frac{1}{k} \hat{\rho}_k,$$

where $\hat{\rho}_k$ is the k -th correlation associated with the residuals after (parametrically) estimating x_t . The matrix A contains the relevant terms of the expression $E_0[\frac{\partial L}{\partial \eta} \frac{\partial L}{\partial \eta'}]$. Its form depends upon the ARMA components. For instance, in the case where $\Phi(L) = \dot{\Theta}(L) = 1$, it becomes

$$A = t_0 \sum_{i=1}^{t_0-1} \frac{1}{i^2} \left(1 - \frac{1}{it_0} \right).$$

It can be easily shown that for any fixed ω , $LM(\omega) \xrightarrow{w} \chi_1^2$. But since ω is, in general, unknown, we adopt a common method used in this scenario and consider test statistics of the form $\sup_{\omega \in \Omega} LM(\omega)$. Critical values can be found in Mayoral (2005). To carry out the test on our data set, residuals are computed using Sowell's ML method.

The first column of table 10 presents the results of the test while the second column displays the date of the break for the cases where it turned out to be significant. It is noteworthy that the results are very homogeneous across countries: for eighteen out of the twenty-one countries no evidence of a change in persistence has been found. That conclusion is only reversed for Austria, Belgium, and Germany, for which some evidence of a break in persistence is found. For all three countries, the shock is found at the beginning of the 1960s. Nevertheless, we should remember that we are running twenty-one tests at the 5 percent significance level and, therefore, we should expect some rejections even if the null hypothesis is true.

In short, our results agree with the recent literature that finds little empirical evidence supporting a change in inflation persistence.

6. Conclusions

This paper explores the inflation rates of a group of OECD countries, focusing on their persistence properties. We propose modeling this data set using ARFIMA models, since they are very flexible, to represent the medium- and long-run properties of time series. An economic justification for the existence of fractionally integrated behavior in the data, as well as solid empirical evidence supporting this hypothesis, is provided. In agreement with previous works, we find that inflation rates are very persistent but, in contrast to most

Table 10. Changes in Persistence

	supLM	Break Date
AU	6.026	—
AUS	25.601**	1964:1
BE	13.373*	1966:1
CA	1.759	—
DK	2.382	—
FI	2.004	—
FR	5.670	—
GE	10.340*	1963:2
GR	3.294	—
IT	1.738	—
JP	0.000	—
LX	6.033	—
NL	3.787	—
NZ	0.761	—
NO	3.270	—
PO	1.451	—
SP	6.850	—
SWE	2.060	—
SWI	1.106	—
UK	3.691	—
USA	4.577	—
C.V.	(9.68,13.5)	—

of them, we believe that shocks do not have, in general, a permanent effect, implying that the series are mean reverting. The latter finding is very relevant since it implies that the $I(1)$ characterization is not suitable for this data set. We have shown that some widely used tools to measure persistence and to test its stability, such as the sum of the AR coefficients (or its equivalent, the cumulative impulse response) are not suitable if the DGP is FI. Since there is always uncertainty about the true DGP, these conclusions should always be taken into account when computing these tools.

Our measures of persistence allow us to establish cross-country comparisons, and it is shown that important differences arise between

the nations that we have considered, which may be related to the different monetary institutions present in each of them. Finally, for most countries, little evidence in favor of a change in inflation persistence has been found, in accordance with the recent literature in this area.

Appendix. Evolution of the Inflation Series

Figure 5. Evolution of Inflation Rates in OECD Countries

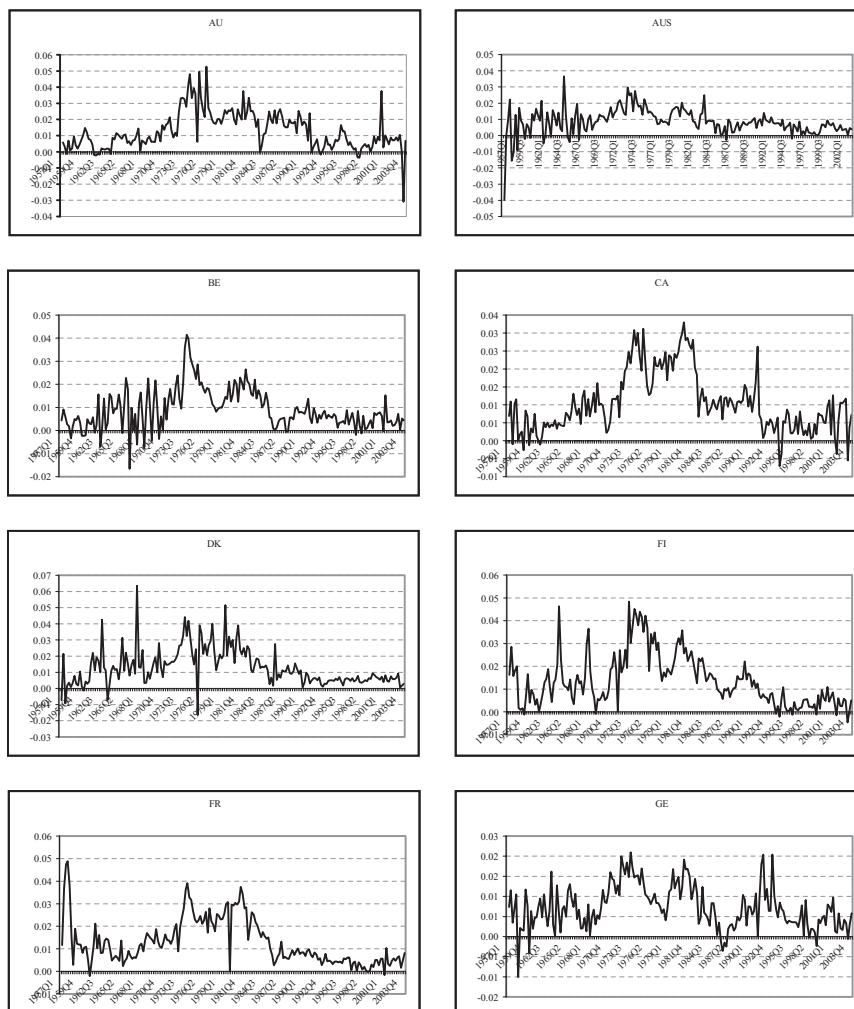


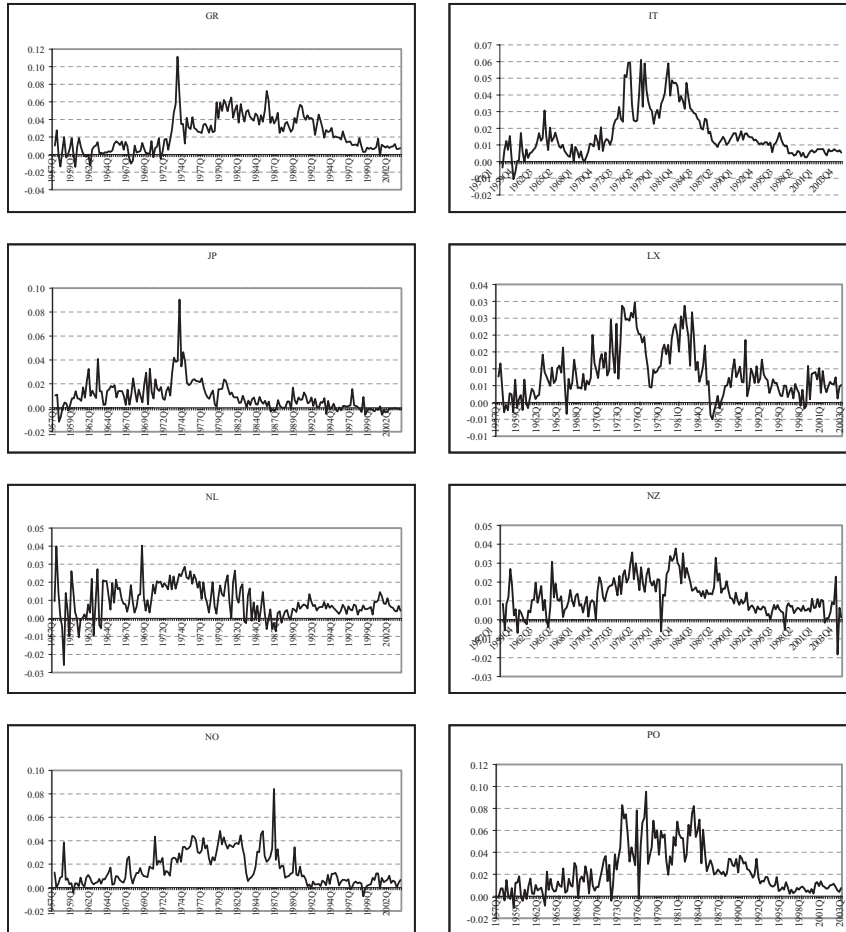
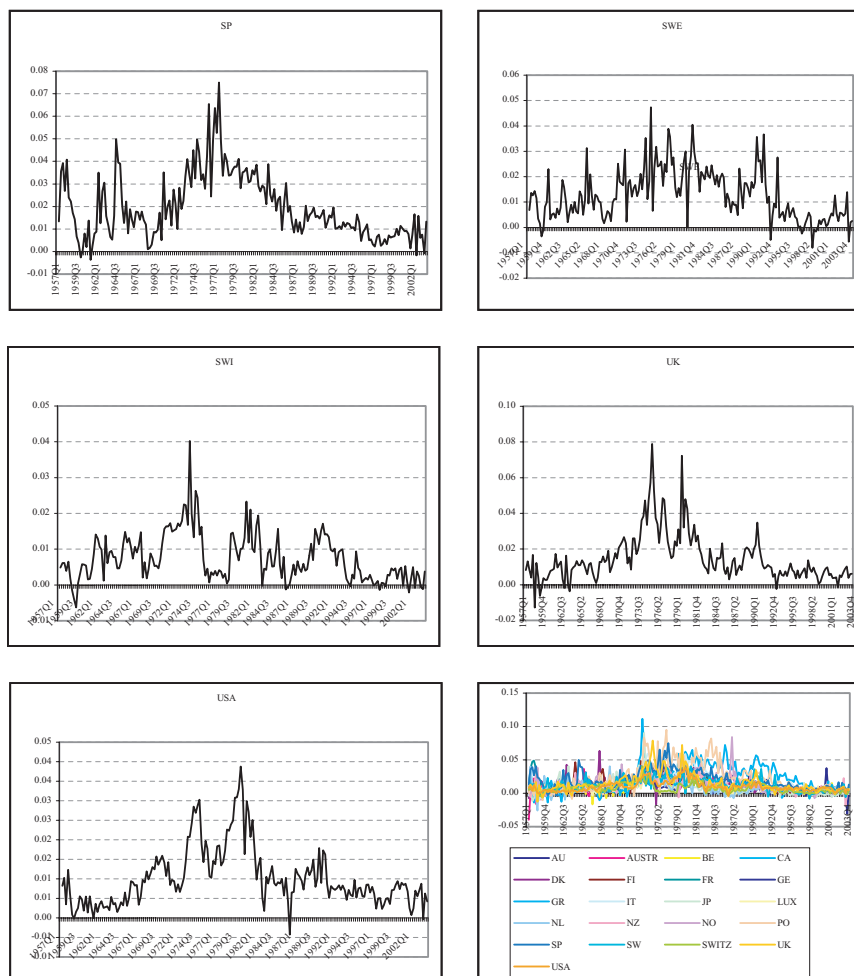
Figure 6. Evolution of Inflation Rates in OECD Countries

Figure 7. Evolution of Inflation Rates in OECD Countries

References

- Abadir, Karim, and Gabriel Talmain. 2002. "Aggregation, Persistence and Volatility in a Macro Model." *Review of Economic Studies* 69 (4): 749–79.
- Agiakloglou, Christos, Paul Newbold, and Mark Wohar. 1992. "Bias in an Estimator of the Fractional Difference Parameter." *Journal of Time Series Analysis* 14 (3): 235–46.

- Alogoskoufis, George. 1992. "Monetary Accommodation, Exchange Rate Regimes and Inflation Persistence." *The Economic Journal* 102 (410): 461–80.
- Alogoskoufis, George, and Ronald Smith. 1991. "The Phillips Curve, the Persistence of Inflation, and the Lucas Critique: Evidence from Exchange-Rate Regimes." *American Economic Review* 81 (5): 1254–75.
- Andersen, Torben G., and Tim Bollerslev. 1997. "Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Frequency Returns." *Journal of Finance* 52 (3): 975–1005.
- Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Paul Labys. 1999. "The Distribution of Exchange Rate Volatility." NBER Working Paper No. 6961.
- Andrews, Donald W.K. 1991. "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica* 59 (3): 817–58.
- . 1993. "Exactly Median-Unbiased Estimation of First Order Estimation of Autoregressive Unit Root Models." *Econometrica* 61 (1): 139–65.
- Andrews, Donald W.K., and Hong-Yuan Chen. 1994. "Approximately Median-Unbiased Estimation of Autoregressive Models." *Journal of Business and Economic Statistics* 12:187–204.
- Angeloni, Ignazio, Günter Coenen, and Frank Smets. 2003. "Persistence, the Transmission Mechanism and Robust Monetary Policy." Working Paper No. 250, European Central Bank.
- Bai, Jushan, and Pierre Perron. 1998. "Estimating and Testing Linear Models with Multiple Structural Changes." *Econometrica* 66 (1): 47–78.
- . 2003a. "Computation and Analysis of Multiple Structural Change Models." *Journal of Applied Econometrics* 18 (1): 1–22.
- . 2003b. "Critical Values for Multiple Structural Change Tests." *The Econometrics Journal* 6 (1): 72–78.
- Baillie Richard T., Ching-Fan Chung, and Margie A. Tieslau. 1992. "The Long Memory and Variability of Inflation: A Reappraisal of the Friedman Hypothesis." Discussion Paper, Tilburg University.
- . 1996. "Analyzing Inflation by the Fractionally Integrated ARFIMA-GARCH Model." *Journal of Applied Econometrics* 11 (1): 23–40.

- Barkoulas, John T., Christopher Baum, and Gurkan S. Oguz. 1998. "Stochastic Long Memory in Traded Goods Prices." *Applied Economics Letters* 5 (3): 135–38.
- Barsky, Robert B. 1987. "The Fisher Hypothesis and the Forecastability and Persistence of Inflation." *Journal of Monetary Economics* 19 (1): 3–24.
- Batini, Nicoletta. 2002. "Euro Area Inflation Persistence." Working Paper No. 201, European Central Bank.
- Baum, Christopher F., John T. Barkoulas, and Mustafa Caglayan. 1999. "Persistence in International Inflation Rates." *Southern Economic Journal* 65 (4): 900–14.
- Benati, Luca. 2002. "Investigating Inflation Persistence across Monetary Regimes I: Empirical Evidence." Mimeo, Bank of England.
- Beran, Jan. 1994. *Statistics for Long-Memory Processes*. New York: Chapman & Hall.
- Bhattacharya, R. N., V. K. Gupta, and E. Waymire. 1983. "The Hurst Effect under Trends." *Journal of Applied Probability* 20: 649–62.
- Bordo, Michael D., and Anna J. Schwartz. 1999. "Under What Circumstances, Past and Present, Have International Rescues of Countries in Financial Distress Been Successful?" *Journal of International Money and Finance* 18 (4): 683–708.
- Bos, Charles, Philip Hans Franses, and Marius Ooms. 1999. "Long Memory and Level Shifts: Re-analyzing Inflation Rates." *Empirical Economics* 24 (3): 427–49.
- . 2002. "Inflation, Forecast Intervals and Long Memory Regression Models." *International Journal of Forecasting* 18 (2): 243–64.
- Breidt, F. Jay, Nuno Crato, and Pedro de Lima. 1998. "The Detection and Estimation of Long Memory in Stochastic Volatility." *Journal of Econometrics* 83 (1–2): 325–48.
- Byers, David, James Davidson, and David Peel. 1997. "Modelling Political Popularity: An Analysis of Long-Range Dependence in Opinion Poll Series." *Journal of the Royal Statistical Society, Series A*, 160 (3): 471–90.
- Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–98.
- Campbell, John Y., and N. Gregory Mankiw. 1987. "Are Output Fluctuations Transitory?" *Quarterly Journal of Economics* 102 (4): 857–80.

- Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans. 2001. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." Working Paper No. 2001-08, Federal Reserve Bank of Chicago.
- Coenen, Günter. 2003. "Inflation Persistence and Robust Monetary Policy Design." Working Paper No. 290, European Central Bank.
- Coenen, Günter, and Volker Wieland. 2005. "A Small Estimated Euro Area Model with Rational Expectations and Nominal Rigidities." *European Economic Review* 49 (5): 1081–1104.
- Cogley, Timothy, and Thomas J. Sargent. 2001. "Evolving Post-World War II U.S. Inflation Dynamics." In *NBER Macroeconomics Annual*, ed. Ben S. Bernanke and Kenneth S. Rogoff. Cambridge, MA: MIT Press.
- Delgado, M., and Peter Robinson. 1994. "New Methods for the Analysis of Long-Memory Time Series." *Journal of Forecasting* 13:97–107.
- Dickey, David A., and Wayne A. Fuller. 1981. "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root." *Econometrica* 49 (4): 1057–72.
- Diebold, Francis X., and Glenn D. Rudebusch. 1989. "Long Memory and Persistence in Aggregate Output." *Journal of Monetary Economics* 24 (2): 189–209.
- . 1991. "On the Power of the Dickey-Fuller Tests against Fractional Alternatives." *Economic Letters* 35 (2): 155–60.
- Diebold, Francis X., and Abdelhak S. Senhadji. 1996. "The Uncertain Unit Root in Real GNP: Comment." *American Economic Review* 86 (5): 1291–98.
- Ding, Zhuanxin., Clive W.J. Granger, and Robert F. Engle. 1993. "A Long Memory Property of Stock Market Returns and a New Model." *Journal of Empirical Finance* 1 (1): 83–106.
- Dolado Juan J., Jesús Gonzalo, and Laura Mayoral. 2002. "A Fractional Dickey-Fuller Test for Unit Roots." *Econometrica* 70 (5): 1963–2006.
- . 2003. "Testing for a Unit Root against Fractional Alternatives in the Presence of a Maintained Trend." Mimeo.
- Doornik, Jurgen A., and Marius Ooms. 2001. "A Package for Estimating, Forecasting and Simulating ARFIMA Models: ARFIMA Package 1.01 for Ox." Mimeo, University of Rotterdam.

- Driscoll, John, and Steiner Holden. 2004. "Fairness and Inflation Persistence." *Journal of the European Economic Association* 2 (2): 240–51.
- Franses, Philip Hans, and Marius Ooms. 1997. "A Periodic Long-Memory Model for Quarterly UK Inflation." *International Journal of Forecasting* 13 (1): 117–26.
- Fuhrer, Jeffrey C. 1997. "The (Un)Importance of Forward-Looking Behavior in Price Setting." *Journal of Money, Credit, and Banking* 29 (3): 338–50.
- Fuhrer, Jeffrey, and George Moore. 1995. "Inflation Persistence." *Quarterly Journal of Economics* 110 (1): 127–59.
- Galí, Jordi, and Mark Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44 (2): 195–222.
- Galí, Jordi, Mark Gertler, and J. David López-Salido. 2001. "European Inflation Dynamics." *European Economic Review* 45 (7): 1237–70.
- Geweke, John, and Susan Porter-Hudak. 1983. "The Estimation and Application of Long Memory Time Series Models." *Journal of Time Series Analysis* 4:221–38.
- Giraitis, Liudas, Piotr Kokoszka, and Remigijus Leipus. 2001. "Testing for Long Memory in the Presence of a General Trend." *Journal of Applied Probability* 38 (4): 1033–54.
- Granger, Clive W.J. 1980. "Long Memory Relationships and the Aggregation of Dynamic Models." *Journal of Econometrics* 14 (2): 227–38.
- Granger, Clive W.J., and R. Joyeux. 1980. "An Introduction to Long Memory Series." *Journal of Time Series Analysis* 1:15–30.
- Hall, Robert E. 1999. Comment on "Rethinking the Role of NAIRU in Monetary Policy: Implications of Model Formulation and Uncertainty," by Arturo Estrella and Frederic S. Mishkin. In *Monetary Policy Rules*, ed. John B. Taylor. Chicago: University of Chicago Press.
- Hassler, Uwe, and Jurgen Wolters. 1995. "Long Memory in Inflation Rates: International Evidence." *Journal of Business and Economic Statistics* 13 (1): 37–45.
- Haubrich, Joseph G., and Andrew W. Lo. 2001. "The Sources and Nature of Long-Term Memory in Aggregate Output." *Economic Review* (Q II):15–30, Federal Reserve Bank of Cleveland.

- Hauser, Michael A., Benedikt M. Pötscher, and Erhard Reschenhofer. 1999. "Measuring Persistence in Aggregate Output: ARMA Models, Fractionally Integrated ARMA Models and Nonparametric Procedures." *Empirical Economics* 24 (2): 243–69.
- Henry, Marc, and Paolo Zaffaroni. 2002. "The Long Range Dependence Paradigm for Macroeconomics and Finance." In *Theory and Applications of Long Range Dependence*, ed. Paul Doukhan, George Oppenheim, and Murad S. Taquq. Boston: Birkhäuser.
- Hondroyannis, George, and Sophia Lazaretou. 2004. "Inflation Persistence during Periods of Structural Change: An Assessment using Greek Data." Working Paper No. 370, European Central Bank.
- Hosking, Jonathan R.M. 1981. "Fractional Differencing." *Biometrika* 68 (1): 165–76.
- Hsu, Chih-Chiang. 2000. "Long Memory or Structural Change: Testing Method and Empirical Examination." Paper contributed to the Econometric Society World Congress, Seattle, WA.
- Jordà, Òscar. 2005. "Estimation and Inference of Impulse Responses by Local Projections." *American Economic Review* 95 (1): 161–82.
- Kim, Jae-Young. 2000. "Detection of Change in Persistence of a Linear Time Series." *Journal of Econometrics* 95 (1): 97–116.
- Kim, Chang-Jin, Charles R. Nelson, and Jeremy Piger. 2004. "The Less-Volatile U.S. Economy: A Bayesian Investigation of Timing, Breadth, and Potential Explanations." *Journal of Business and Economic Statistics* 22 (1): 80–93.
- Koop, Gary, Eduardo Ley, Jacek Osiewalski, and Mark F.J. Steel. 1997. "Bayesian Analysis of Long Memory and Persistence using ARFIMA Models." *Journal of Econometrics* 76 (1–2): 149–69.
- Krämer, Walter, and Philipp Sibbertsen. 2002. "Testing for Structural Change in the Presence of Long Memory." *International Journal of Business and Economics* 1 (3): 235–43.
- Künsh, H. 1986. "Discrimination between Deterministic Trends and Long Range Dependence." *Journal of Applied Probability* 23: 1025–30.
- Kwiatkowski, Denis, Peter C.B. Phillips, Peter Schmidt, and Yongcheol Shin. 1992. "Testing the Null Hypothesis of

- Stationarity against the Alternative of a Unit Root." *Journal of Econometrics* 54 (1–3): 159–78.
- Lee, Dongin, and Peter Schmidt. 1996. "On the Power of the KPSS Test of Stationarity against Fractionally-Integrated Alternatives." *Journal of Econometrics* 73 (1): 285–302.
- Levin, Andrew, and Jeremy M. Piger. 2003. "Is Inflation Persistence Intrinsic in Industrial Economies?" Working Paper No. 023E, Federal Reserve Bank of St. Louis.
- Mandelbrot, Benoit, and James R. Wallis. 1969. "Robustness of the Rescaled Range R/S in the Measurement of Noncyclic Long-Run Statistical Dependence." *Water Resources Research* 5:967–88.
- Mayoral, Laura. 2004a. "A New Minimum Distance Estimator for ARFIMA Processes." Mimeo.
- . 2004b. "Is the Observed Persistence Spurious or Real? A Test for Fractional Integration versus Structural Breaks." Mimeo.
- . 2005. "Lagrange Multiplier Tests for Long Memory and Breaks." Mimeo.
- Michelacci, Claudio, and Paolo Zaffaroni. 2000. "(Fractional) Betaconvergence." *Journal of Monetary Economics* 45 (1): 129–53.
- Mikosch, Thomas, and Catalin Starica. 2004. "Non-stationarities in Financial Time Series, the Long Range Dependence, and the IGARCH Effects." *Review of Economics and Statistics* 86 (1): 378–90.
- Newey, Whitney K., and Kenneth D. West. 1994. "Automatic Lag Selection in Covariance Matrix Estimation." *Review of Economics Studies* 61:631–53.
- Ng, Serena, and Pierre Perron. 2001. "Lag Length Selection and the Construction of Unit Root Tests with Good Size and Power." *Econometrica* 69 (6): 1519–54.
- Nunes, Luis C., Chung-Ming Kuan, and Paul Newbold. 1995. "Spurious Break." *Econometric Theory* 11 (4): 736–49.
- Ooms, Marius, and Jurgen A. Doornik. 1999. "Inference and Forecasting for Fractional Autoregressive Integrated Moving Average Models; with an Application to US and UK Inflation." *Econometric Institute Report 9947/A*, Erasmus University Rotterdam.
- O'Reilly, Gerard, and Karl Whelan. 2004. "Has Euro-Area Inflation Persistence Changed over Time?" Working Paper No. 335, European Central Bank.

- Parke, William R. 1999. "What Is Fractional Integration?" *The Review of Economics and Statistics* 81 (4): 632–38.
- Perron, Pierre. 1989. "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis." *Econometrica* 58:1361–1401.
- Perron, Pierre, and Xiaokang Zhu. 2005. "Structural Breaks with Deterministic and Stochastic Trends." *Journal of Econometrics* 129 (1–2): 65–119.
- Phillips, Peter C.B., and Pierre Perron. 1988. "Testing for a Unit Root in Time Series Regression." *Biometrika* 75 (2): 335–46.
- Pivetta, Frederic, and Ricardo Reis. 2004. "The Persistence of Inflation in the United States." Mimeo, Harvard University.
- Roberts, John M. 2001. "How Well Does the New Keynesian Sticky-Price Model Fit the Data?" Finance and Economics Discussion Series Paper No. 2001-13, Board of Governors of the Federal Reserve System.
- Robinson, Peter M. 1978. "Statistical Inference for a Random Coefficient Autoregressive Model." *Scandinavian Journal of Statistics* 5:163–68.
- Rotemberg, Julio. 1987. "The New Keynesian Microfoundations." *Macroeconomics Annual* 2:69–104.
- Sargent, Thomas J. 1999. *The Conquest of American Inflation*. Princeton, NJ: Princeton University Press.
- Solow, R. 1968. "Recent Controversy on the Theory of Inflation: An Eclectic View." In *Inflation: Its Causes, Consequences, and Control; Proceedings of a Symposium*, ed. Stephen W. Rousseas. New York: New York University.
- Sowell, Fallaw. 1992a. "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series." *Journal of Econometrics* 53 (1–3): 165–88.
- . 1992b. "Modeling Long-Run Behavior with the Fractional ARIMA Model." *Journal of Monetary Economics* 29 (2): 277–302.
- Stock, J. 2001. "Comment on Evolving Post World War II U.S. Inflation Dynamics." Mimeo, Harvard University.
- Taylor, John B. 1979. "Staggered Wage Setting in a Macro Model." *American Economic Review* 69:108–13.
- . 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (1): 1–23.

- . 1998. “Monetary Policy Guidelines for Unemployment and Inflation Stability.” In *Inflation, Unemployment and Monetary Policy*, ed. Robert M. Solow and John B. Taylor. Cambridge, MA: MIT Press.
- . 2000. “Low Inflation, Pass-Through and the Pricing Power of Firms.” *European Economic Review* 44 (7): 1389–1408.
- Teverosky, V., and M. Taqqu. 1997. “Testing for Long-Range Dependence in the Presence of Shifting Means or a Slowly Declining Trend, Using a Variance-Type Estimator.” *Journal of Time Series Analysis* 18 (3): 279–304.
- Tobin, J. 1968. “Discussion.” In *Inflation: Its Causes, Consequences, and Control; Proceedings of a Symposium*, ed. Stephen W. Rousseas. New York: New York University.
- Zaffaroni, Paolo. 2004. “Contemporaneous Aggregation of Linear Dynamic Models in Large Economies.” *Journal of Econometrics* 120 (1): 75–102.